

# IS 2150 / TEL 2810

## Information Security & Privacy



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Access Control Model  
Foundational Results

Lecture 3  
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# Objective

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- Understand the basic results of the HRU model
  - Safety issue
  - Turing machine
  - Undecidability



# Protection System

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- State of a system
  - Current values of
    - memory locations, registers, secondary storage, etc.
    - other system components
- Protection state (P)
  - A system state that is considered secure
- A protection system
  - Captures the conditions for state transition
  - Consists of two parts:
    - A set of generic rights
    - A set of commands



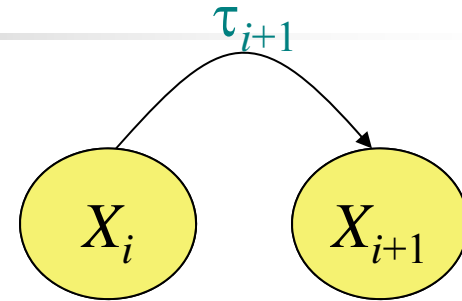
# Protection System

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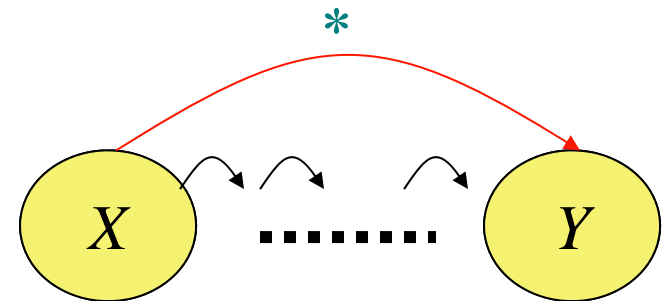
- Subject ( $S$ : set of all subjects)
  - Eg.: users, processes, agents, etc.
- Object ( $O$ : set of all objects)
  - Eg.: Processes, files, devices
- Right ( $R$ : set of all rights)
  - An action/operation that a subject is allowed/disallowed on objects
  - Access Matrix  $A$ :  $a[s, o] \subseteq R$
- Set of Protection States:  $(S, O, A)$ 
  - Initial state  $X_0 = (S_0, O_0, A_0)$

# State Transitions

$X_i \xrightarrow{\tau_{i+1}} X_{i+1}$  : upon transition  $\tau_{i+1}$ , the system moves from state  $X_i$  to  $X_{i+1}$

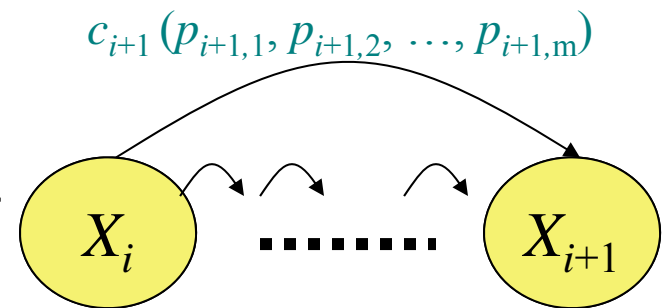


$X \xrightarrow{*} Y$  : the system moves from state  $X$  to  $Y$  after a set of transitions



$X_i \xrightarrow{c_{i+1}(p_{i+1,1}, p_{i+1,2}, \dots, p_{i+1,m})} X_{i+1}$  : state transition upon a command

For every command there is a sequence of state transition operations





# Primitive commands (HRU)

Create subject $s$	Creates new row, column in ACM; $s$ does not exist prior to this
Create object $o$	Creates new column in ACM $o$ does not exist prior to this
Enter $r$ into $a[s, o]$	Adds $r$ right for subject $s$ over object $o$ Ineffective if $r$ is already there
Delete $r$ from $a[s, o]$	Removes $r$ right from subject $s$ over object $o$
Destroy subject $s$	Deletes row, column from ACM;
Destroy object $o$	Deletes column from ACM



# Primitive commands (HRU)

---

Create subject  $s$

Creates new row, column in ACM;  
 $s$  does not exist prior to this

Precondition:  $s \notin S$

Postconditions:

$$S' = S \cup \{s\}, O' = O \cup \{s\}$$

$(\forall y \in O')[a'[s, y] = \emptyset]$  (row entries for  $s$ )

$(\forall x \in S')[a'[x, s] = \emptyset]$  (column entries for  $s$ )

$(\forall x \in S)(\forall y \in O)[a'[x, y] = a[x, y]]$



# Primitive commands (HRU)

---

Enter  $r$  into  $a[s, o]$

Adds  $r$  right for subject  $s$  over object  $o$

Ineffective if  $r$  is already there

Precondition:  $s \in S, o \in O$

Postconditions:

$$S' = S, O' = O$$

$$a'[s, o] = a[s, o] \cup \{r\}$$

$$(\forall x \in S')(\forall y \in O')$$

$$[(x, y) \neq (s, o) \rightarrow a'[x, y] = a[x, y]]$$





# System commands

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- [Unix] process  $p$  creates file  $f$  with owner  $own$  and  $read$  and  $write$  ( $r, w$ ) will be represented by the following:

Command  $create\_file(p, f)$

Create object  $f$

Enter  $own$  into  $a[p, f]$

Enter  $r$  into  $a[p, f]$

Enter  $w$  into  $a[p, f]$

End



# System commands

---

- Process  $p$  creates a new process  $q$

Command *spawn\_process*( $p, q$ )

Create subject  $q$ ;

Enter *own* into  $a[p, q]$

Enter  $r$  into  $a[p, q]$

Enter  $w$  into  $a[p, q]$

Enter  $r$  into  $a[q, p]$

Enter  $w$  into  $a[q, p]$

End

← Parent and child can  
signal each other



# System commands

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- Defined commands can be used to update ACM

Command *make\_owner(p, f)*

Enter *own* into *a[p,f]*

End

- Mono-operational:
  - the command invokes only one primitive



# Conditional Commands

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- Mono-operational + mono-conditional

Command *grant\_read\_file*( $p, f, q$ )

  If *own* in  $a[p, f]$

  Then

    Enter  $r$  into  $a[q, f]$

End

# Conditional Commands

- Mono-operational + biconditional

Command *grant\_read\_file*(*p*, *f*, *q*)

If *r* in *a*[*p*,*f*] and *c* in *a*[*p*,*f*]

Then

Enter *r* into *a*[*q*,*f*]

End

- Why not "OR"??

Command *grant\_read\_file*(*p*, *f*, *q*)  
If *r* in *a*[*p*,*f*] OR *c* in *a*[*p*,*f*]  
Then  
Enter *r* into *a*[*q*,*f*]  
End

Command *grant\_read\_file1*(*p*, *f*, *q*)  
If *r* in *a*[*p*,*f*]  
Then  
Enter *r* into *a*[*q*,*f*]  
End

Command *grant\_read\_file2*(*p*, *f*, *q*)  
If *c* in *a*[*p*,*f*]  
Then  
Enter *r* into *a*[*q*,*f*]  
End



Executing command:  
***grant\_read\_file***  
is equivalent to executing  
commands:  
***grant\_read\_file1*;**  
***grant\_read\_file2***



# Fundamental questions

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- How can we determine that a system is secure?
  - Need to define what we mean by a system being “secure”
- Is there a generic algorithm that allows us to determine whether a computer system is secure?



# What is a secure system?

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- A simple definition
  - A secure system doesn't allow violations of a security policy
- Alternative view: based on distribution of rights
  - **Leakage of rights:** (unsafe with respect to right  $r$ )
    - Assume that  $A$  representing a secure state does not contain a right  $r$  in an element of  $A$ .
    - A right  $r$  is said to be leaked, if a sequence of operations/commands adds  $r$  to an element of  $A$ , which did not contain  $r$



# What is a secure system?

---

- Safety of a system with initial protection state  $X_0$ 
  - Safe with respect to  $r$ : System is *safe with respect to  $r$*  if  $r$  can never be leaked
  - Else it is called *unsafe with respect to right  $r$* .





# Safety Problem: *formally*

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- Given
  - Initial state  $X_0 = (S_0, O_0, A_0)$
  - Set of primitive commands  $c$
  - $r$  is not in  $A_0[s, o]$
- Can we reach a state  $X_n$  where
  - $\exists s, o$  such that  $A_n[s, o]$  includes a right  $r$  not in  $A_0[s, o]$ ?
    - If so, the system is not safe
    - But is "safe" secure?



# Undecidable Problems

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- Decidable Problem
  - A decision problem can be solved by an algorithm that halts on all inputs in a finite number of steps.
- Undecidable Problem
  - A problem that cannot be solved for all cases by any algorithm whatsoever

# Decidability Results

*(Harrison, Ruzzo, Ullman)*

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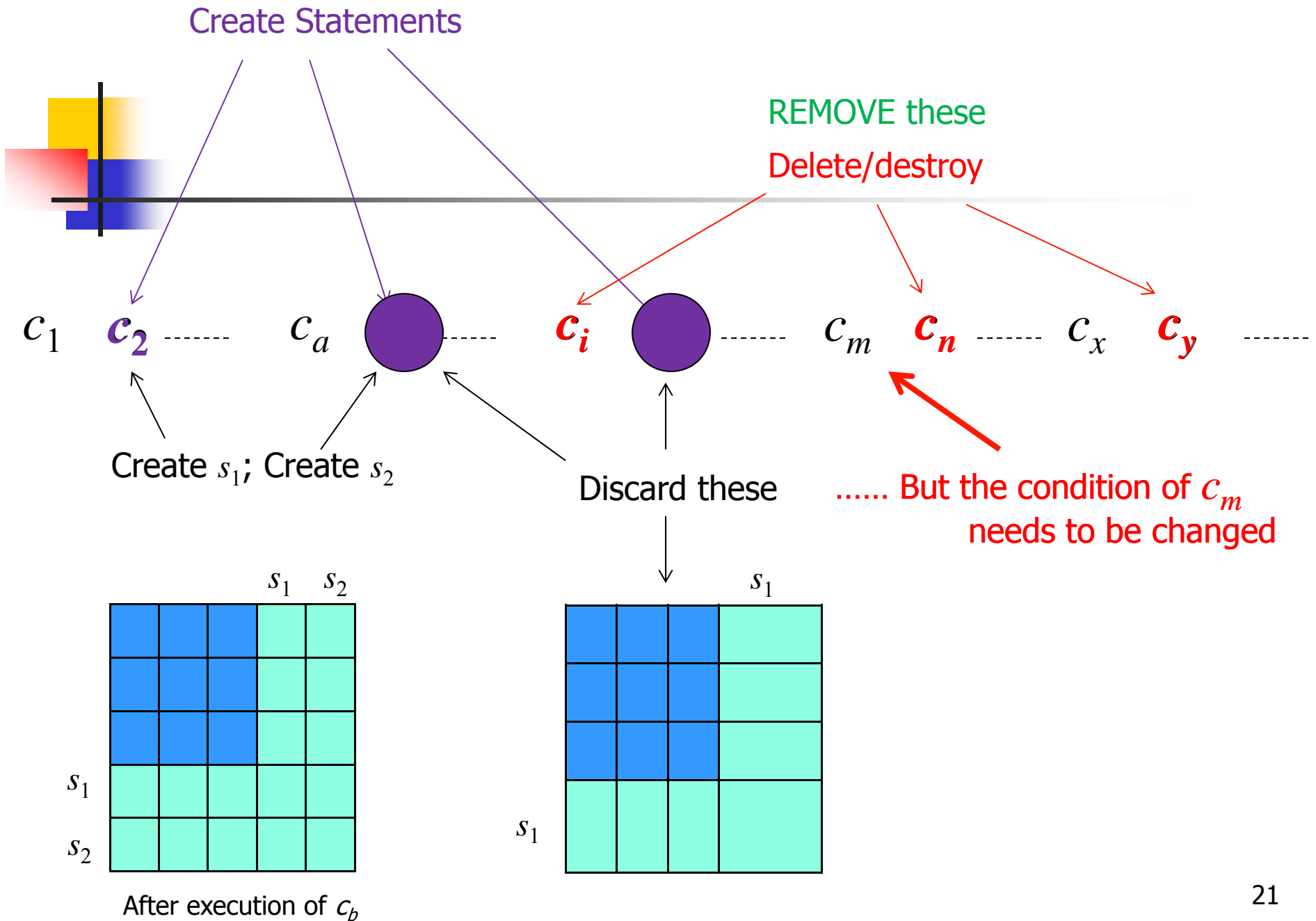
- Theorem:

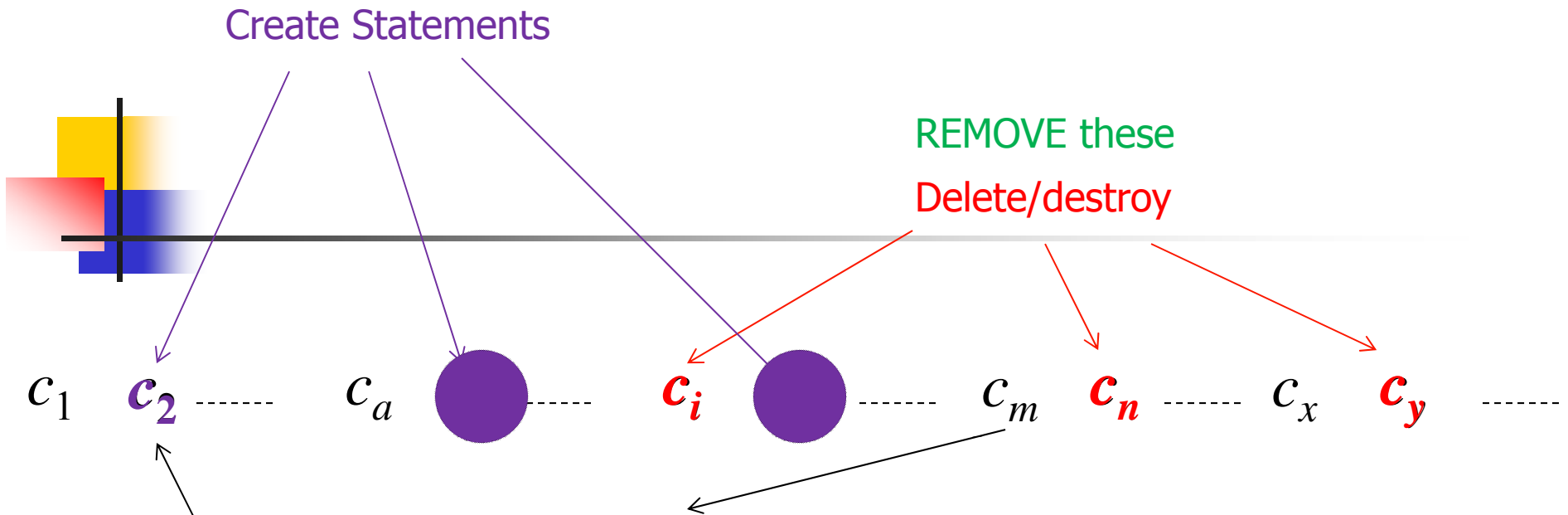
- Given a system where each command consists of a single *primitive* command (mono-operational), there exists an algorithm that will determine if a protection system with initial state  $X_0$  is safe with respect to right  $r$ .

# Decidability Results

*(Harrison, Ruzzo, Ullman)*

- Proof: determine minimum commands  $k$  to leak
  - Delete/destroy: Can't leak
  - Create/enter: new subjects/objects "equal", so treat all new subjects as one
    - No test for absence of right
    - Tests on  $A[s_1, o_1]$  and  $A[s_2, o_2]$  have same result as the same tests on  $A[s_1, o_1]$  and  $A[s_1, o_2] = A[s_1, o_2] \cup A[s_2, o_2]$
  - If  $n$  rights leak possible, must be able to leak  $k = n(|S_o| + 1)(|O_o| + 1) + 1$  commands
  - Enumerate all possible states to decide





Create  $s_1$

If *Condition*  
Enter statement

	$o_1$	$o_2$	$s_1$	$s_2$
	Blue	Blue	Light Green	Light Green
	Blue	Blue	Light Green	Light Green
	Blue	Blue	Light Green	Light Green
$s_1$	Light Green	X Y	Light Green	Light Green
$s_2$	Light Green	Z	Light Green	Light Green

After two creates

$r \in A[s_1, o_1]$

→

$r \in A[s_1, o_1]$

$r \in A[s_2, o_2]$

→

$r \in A[s_1, o_2]$   
where  $A[s_1, o_2] =$   
 $A[s_1, o_2] \cup A[s_2, o_2]$

	$o_1$	$o_2$	$s_1$
	Blue	Blue	Light Green
	Blue	Blue	Light Green
	Blue	Blue	Light Green
$s_1$	Light Green	X Y ∪ Z	Light Green

Just use first create

# Decidability Results

*(Harrison, Ruzzo, Ullman)*

- Proof: determine minimum commands  $k$  to leak
  - Delete/destroy: Can't leak
  - Create/enter: new subjects/objects "equal", so treat all new subjects as one
    - No test for absence of right
    - Tests on  $A[s_1, o_1]$  and  $A[s_2, o_2]$  have same result as the same tests on  $A[s_1, o_1]$  and  $A[s_1, o_2] = A[s_1, o_2] \cup A[s_2, o_2]$
  - If  $n$  rights leak possible, must be able to leak  $k = n(|S_o| + 1)(|O_o| + 1) + 1$  commands
  - Enumerate all possible states to decide

# Decidability Results

*(Harrison, Ruzzo, Ullman)*

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- It is undecidable if a given state of a given protection system is safe for a given generic right
- For proof – need to know Turing machines and halting problem





# Turing Machine & halting problem

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- **The halting problem:**
  - Given a description of an algorithm and a description of its initial arguments, determine whether the algorithm, when executed with these arguments, ever halts (the alternative is that it runs forever without halting).



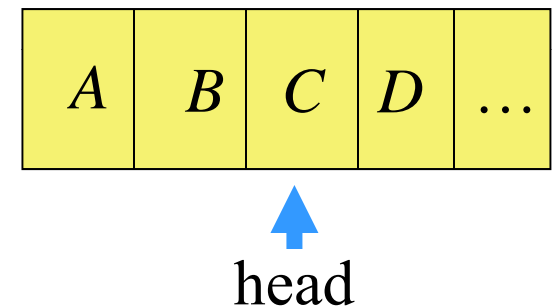
# Turing Machine & Safety problem

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- Theorem:
  - It is undecidable if a given state of a given protection system is safe for a given generic right
- Reduce TM to Safety problem
  - If Safety problem is decidable then it implies that TM halts (for all inputs) – showing that the halting problem is decidable (contradiction)
- TM is an abstract model of computer
  - Alan Turing in 1936

# Turing Machine

- TM consists of
  - A tape divided into cells; infinite in one direction
  - A set of tape symbols  $M$ 
    - $M$  contains a special blank symbol  $b$
  - A set of states  $K$
  - A head that can read and write symbols
  - An action table that tells the machine how to transition
    - What symbol to write
    - How to move the head ('L' for left and 'R' for right)
    - What is the next state

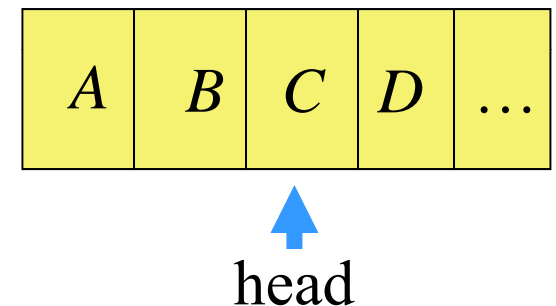


Current state is  $k$

Current symbol is  $C$

# Turing Machine

- Transition function  $\delta(k, m) = (k', m', L)$ :
  - In state  $k$ , symbol  $m$  on tape location is replaced by symbol  $m'$ ,
  - Head moves one cell to the left, and TM enters state  $k'$
- Halting state is  $q_f$ 
  - TM halts when it enters this state

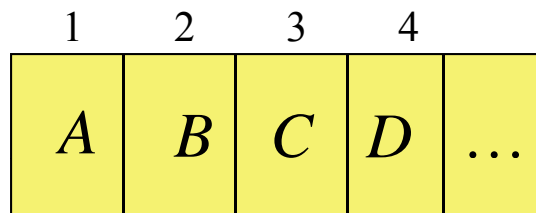


Current state is  $k$

Current symbol is  $C$

Let  $\delta(k, C) = (k_1, X, R)$   
where  $k_1$  is the next state

# Turing Machine

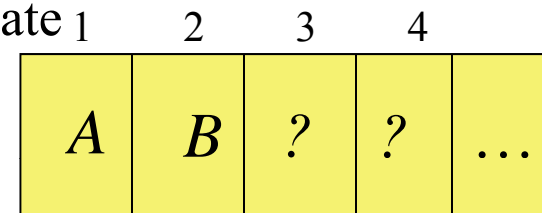
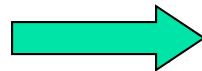


↑  
head

Current state is  $k$   
Current symbol is  $C$

Let  $\delta(k, C) = (k_1, X, R)$

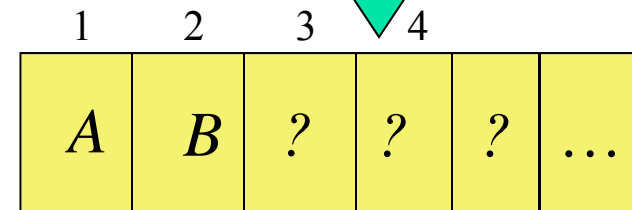
where  $k_1$  is the next state



↑  
head

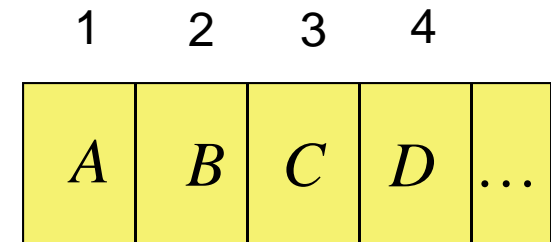
Let  $\delta(k_1, D) = (k_2, Y, L)$

where  $k_2$  is the next state



↑  
?  
head

# TM2Safety Reduction



Current state is  $k$

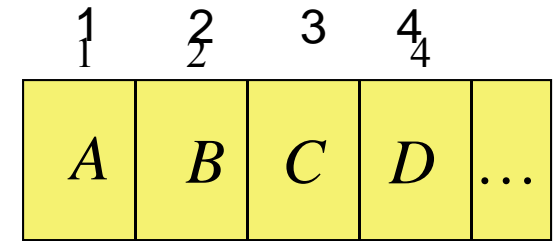
Current symbol is  $C$  head

Proof: Reduce TM to safety problem

- Symbols, States  $\Rightarrow$  rights
- Tape cell  $\Rightarrow$  subject
- Cell  $s_i$  has  $A \Rightarrow s_i$  has  $A$  rights on itself
- Cell  $s_k \Rightarrow s_k$  has end rights on itself
- State  $p$ , head at  $s_i \Rightarrow s_i$  has  $p$  rights on itself
- Distinguished Right *own*:
  - $s_i$  owns  $s_{i+1}$  for  $1 \leq i < k$

	$s_1$	$s_2$	$s_3$	$s_4$	
$s_1$	A	<i>own</i>			
$s_2$		B	<i>own</i>		
$s_3$			C $k$	<i>own</i>	
$s_4$				D end	

# Command Mapping (Left move)



Current state is  $k$

Current symbol is  $C$  ↑  
head

$$\delta(k, C) = (k_1, X, L)$$

$$\delta(k, C) = (k_1, X, L)$$

*If head is not in leftmost*

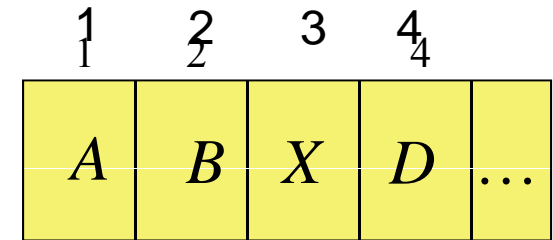
command  $c_{k,C}(s_i, s_{i-1})$   
 if  $own$  in  $a[s_{i-1}, s_i]$  and  $k$  in  
 $a[s_i, s_i]$  and  $C$  in  $a[s_i, s_i]$   
 then

delete  $k$  from  $a[s_i, s_i]$ ;  
 delete  $C$  from  $a[s_i, s_i]$ ;  
 enter  $X$  into  $a[s_i, s_i]$ ;  
 enter  $k_1$  into  $a[s_{i-1}, s_{i-1}]$ ;

End

	$s_1$	$s_2$	$s_3$	$s_4$	
$s_1$	A	$own$			
$s_2$		B	$own$		
$s_3$			$C k$	$own$	
$s_4$				D end	

# Command Mapping (Left move)



Current state is  $k_1$

Current symbol is  $D$  head

$$\delta(k, C) = (k_1, X, L)$$

$$\delta(k, C) = (k_1, X, L)$$

*If head is not in leftmost*

command  $c_{k,C}(s_i, s_{i-1})$   
 if  $own$  in  $a[s_{i-1}, s_i]$  and  $k$  in  
 $a[s_i, s_i]$  and  $C$  in  $a[s_i, s_i]$   
 then

delete  $k$  from  $a[s_i, s_i]$ ;  
 delete  $C$  from  $a[s_i, s_i]$ ;  
 enter  $X$  into  $a[s_i, s_i]$ ;  
 enter  $k_1$  into  $a[s_{i-1}, s_{i-1}]$ ;

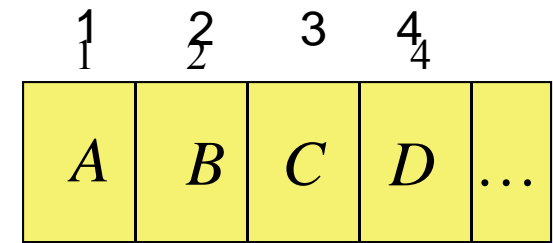
End

If head is in leftmost both  $s_i$  and  $s_{i-1}$  are  $s_1$

	$s_1$	$s_2$	$s_3$	$s_4$	
$s_1$	A	$own$			
$s_2$		B $k_1$	$own$		
$s_3$			X	$own$	
$s_4$				D end	



# Command Mapping (Right move)



Current state is  $k$

Current symbol is  $C$

↑  
head

$$\delta(k, C) = (k_1, X, R)$$

$$\delta(k, C) = (k_1, X, R)$$

command  $c_{k,C}(s_i, s_{i+1})$   
 if  $own$  in  $a[s_i, s_{i+1}]$  and  $k$   
 in  $a[s_i, s_i]$  and  $C$  in  
 $a[s_i, s_i]$   
 then  
 delete  $k$  from  $a[s_i, s_i]$ ;  
 delete  $C$  from  $a[s_i, s_i]$ ;  
 enter  $X$  into  $a[s_i, s_i]$ ;  
 enter  $k_1$  into  $a[s_{i+1},$   
 $s_{i+1}]$ ;  
 end

	$s_1$	$s_2$	$s_3$	$s_4$	
$s_1$	A	<i>own</i>			
$s_2$		B	<i>own</i>		
$s_3$			$C k$	<i>own</i>	
$s_4$				D end	

# Command Mapping (Right move)

1	2	3	4	
A	B	X	D	...

Current state is  $k_1$

Current symbol is C

↑  
head

$$\delta(k, C) = (k_1, X, R)$$

$$\delta(k, C) = (k_1, X, R)$$

command  $c_{k,C}(s_i, s_{i+1})$   
 if  $own$  in  $a[s_i, s_{i+1}]$  and  $k$   
 in  $a[s_i, s_i]$  and C in  
 $a[s_i, s_i]$   
 then  
 delete  $k$  from  $a[s_i, s_i]$ ;  
 delete C from  $a[s_i, s_i]$ ;  
 enter X into  $a[s_i, s_i]$ ;  
 enter  $k_1$  into  $a[s_{i+1},$   
 $s_{i+1}]$ ;  
 end

	$s_1$	$s_2$	$s_3$	$s_4$	
$s_1$	A	<i>own</i>			
$s_2$		B	<i>own</i>		
$s_3$			X	<i>own</i>	
$s_4$				D $k_1$ end	

# Command Mapping (Rightmost move)

1	2	3	4	
A	B	X	D	...

Current state is  $k_1$

Current symbol is C

↑  
head

$\delta(k_1, D) = (k_2, Y, R)$  at end becomes

$\delta(k_1, C) = (k_2, Y, R)$

command  $\text{crightmost}_{k,C}(s_i, s_{i+1})$   
 if *end* in  $a[s_i, s_i]$  and  $k_1$  in  $a[s_i, s_i]$   
 and D in  $a[s_i, s_i]$

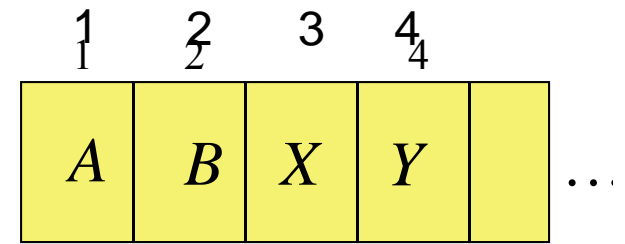
then

delete *end* from  $a[s_i, s_i]$ ;  
 create subject  $s_{i+1}$ ;  
 enter *own* into  $a[s_i, s_{i+1}]$ ;  
 enter *end* into  $a[s_{i+1}, s_{i+1}]$ ;  
 delete  $k_1$  from  $a[s_i, s_i]$ ;  
 delete D from  $a[s_i, s_i]$ ;  
 enter Y into  $a[s_i, s_i]$ ;  
 enter  $k_2$  into  $a[s_i, s_i]$ ;

end

	$s_1$	$s_2$	$s_3$	$s_4$	
$s_1$	A	<i>own</i>			
$s_2$		B	<i>own</i>		
$s_3$			X	<i>own</i>	
$s_4$				D $k_1$ end	

# Command Mapping (Rightmost move)



Current state is  $k_1$

Current symbol is  $D$

head

$\delta(k_1, D) = (k_2, Y, R)$  at end becomes

$\delta(k_1, D) = (k_2, Y, R)$

command  $\text{crightmost}_{k,C}(s_i, s_{i+1})$   
 if  $end$  in  $a[s_i, s_i]$  and  $k_1$  in  $a[s_i, s_i]$   
 and  $D$  in  $a[s_i, s_i]$

then

delete  $end$  from  $a[s_i, s_i]$ ;  
 create subject  $s_{i+1}$ ;  
 enter  $own$  into  $a[s_i, s_{i+1}]$ ;  
 enter  $end$  into  $a[s_{i+1}, s_{i+1}]$ ;  
 delete  $k_1$  from  $a[s_i, s_i]$ ;  
 delete  $D$  from  $a[s_i, s_i]$ ;  
 enter  $Y$  into  $a[s_i, s_i]$ ;  
 enter  $k_2$  into  $a[s_i, s_i]$ ;

end

	$s_1$	$s_2$	$s_3$	$s_4$	$s_5$
$s_1$	A	<i>own</i>			
$s_2$		B	<i>own</i>		
$s_3$			X	<i>own</i>	
$s_4$				Y	<i>own</i>
$s_5$					<b>b</b> $k_2$ end



# Rest of Proof

---

- Protection system exactly simulates a TM
  - Exactly 1 *end* right in ACM
  - Only 1 right corresponds to a state
  - Thus, at most 1 applicable command in each configuration of the TM
- If TM enters state  $q_f$  then right has leaked
- If safety question decidable, then represent TM as above and determine if  $q_f$  leaks
  - Leaks halting state  $\Rightarrow$  halting state in the matrix  $\Rightarrow$  Halting state reached
- Conclusion: safety question undecidable



# Other results

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- For protection system without the create primitives, (i.e., delete **create** primitive); the safety question is complete in **P-SPACE**
- It is undecidable whether a given configuration of a given monotonic protection system is safe for a given generic right
  - Delete **destroy**, **delete** primitives;
  - The system becomes monotonic as they only increase in size and complexity
- The safety question for biconditional monotonic protection systems is undecidable
- The safety question for monoconditional, monotonic protection systems is decidable
- The safety question for monoconditional protection systems with **create**, **enter**, **delete** (and no **destroy**) is decidable.



# Summary

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- HRU Model
- Some foundational results showing that guaranteeing security is hard problem