IS 2150 / TEL 2810
Information Security & Privacy

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Access Control Model
Foundational Results

Lecture 3
Sept 16, 2015

Objective

- Understand the basic results of the HRU model
  - Saftey issue
  - Turing machine
  - Undecidability

Protection System

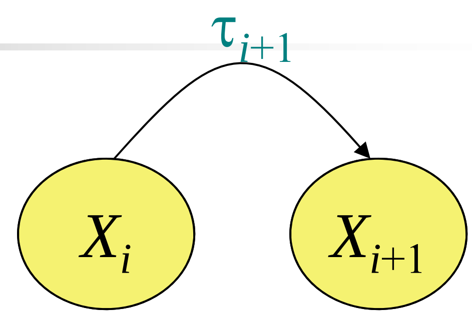
- State of a system
  - Current values of
    - memory locations, registers, secondary storage, etc.
    - other system components
- Protection state (P)
  - A system state that is considered secure
- A protection system
  - Captures the conditions for state transition
  - Consists of two parts:
    - A set of generic rights
    - A set of commands

Protection System

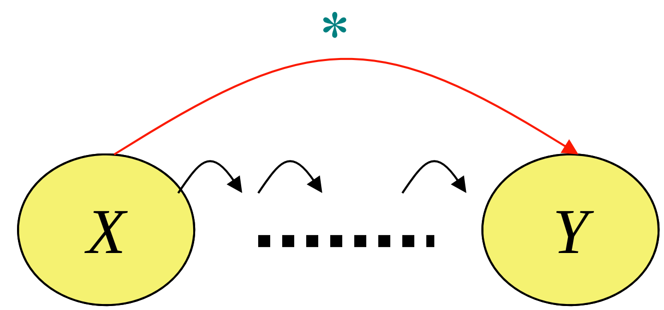
- Subject (S: set of all subjects)
  - Eg.: users, processes, agents, etc.
- Object (O: set of all objects)
  - Eg.:Processes, files, devices
- Right (R: set of all rights)
  - An action/operation that a subject is allowed/disallowed on objects
  - Access Matrix A: $a[s, o] \subseteq R$
- Set of Protection States: (S, O, A)
  - Initial state $X_0 = (S_0, O_0, A_0)$

State Transitions

$X_i \vdash_{\tau_{i+1}} X_{i+1}$: upon transition τ_{i+1}, the system moves from state X_i to X_{i+1}

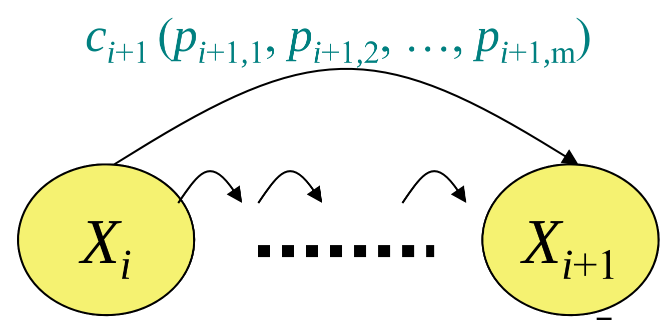


$X \vdash^{*} Y$: the system moves from state X to Y after a set of transitions



$X_i \vdash c_{i+1}\,(p_{i+1,1}, p_{i+1,2}, \ldots, p_{i+1,m})\; X_{i+1}$:
state transition upon a command

- For every command there is a sequence of state transition operations



Primitive commands (HRU)

Create subject s	Creates new row, column in ACM; s does not exist prior to this
Create object o	Creates new column in ACM o does not exist prior to this
Enter r into $a[s, o]$	Adds r right for subject s over object o Ineffective if r is already there
Delete r from $a[s, o]$	Removes r right from subject s over object o
Destroy subject s	Deletes row, column from ACM;
Destroy object o	Deletes column from ACM

Primitive commands (HRU)

Create subject s

Creates new row, column in ACM;
s does not exist prior to this

Precondition: $s \notin S$
Postconditions:

$S' = S \cup \{ s \}$, $O' = O \cup \{ s \}$

$(\forall y \in O')[a'[s, y] = \varnothing]$ (row entries for s)
$(\forall x \in S')[a'[x, s] = \varnothing]$ (column entries for s)
$(\forall x \in S)(\forall y \in O)[a'[x, y] = a[x, y]]$

Primitive commands (HRU)

Enter r into $a[s, o]$

Adds r right for subject s over object o

Ineffective if r is already there

Precondition: $s \in S$, $o \in O$

Postconditions:

$$S' = S, O' = O$$

$$a'[s, o] = a[s, o] \cup \{ r \}$$

$$(\forall x \in S')(\forall y \in O')$$

$$[(x, y) \neq (s, o) \rightarrow a'[x, y] = a[x, y]]$$

System commands

- [Unix] process p creates file f with owner *read* and *write* (r, w) will be represented by the following:

  Command $create\_file(p, f)$
    Create object f
    Enter *own* into $a[p,f]$
    Enter r into $a[p,f]$
    Enter w into $a[p,f]$
  End

System commands

- Process p creates a new process q

  Command $spawn\_process(p, q)$

  Create subject q;

  Enter own into $a[p,q]$

  Enter r into $a[p,q]$

  Enter w into $a[p,q]$

  Enter r into $a[q,p]$

  Enter w into $a[q,p]$

  End

Parent and child can signal each other

System commands

- Defined commands can be used to update ACM

  Command *make_owner*(p, f)

  Enter *own* into $a[p,f]$

  End

- Mono-operational:
  - the command invokes only one primitive

Conditional Commands

- Mono-operational + mono-conditional

Command *grant_read_file*(p, f, q)

If *own* in $a[p,f]$

Then

Enter r into $a[q,f]$

End

Conditional Commands

- Mono-operational + biconditional

Command *grant_read_file*(*p, f, q*)
  If *r* in *a*[*p,f*] and *c* in *a*[*p,f*]
  Then
    Enter *r* into *a*[*q,f*]
End

- Why not "OR"??

Command *grant_read_file*(*p, f, q*)
  If *r* in *a*[*p,f*] OR *c* in *a*[*p,f*]
  Then
    Enter *r* into *a*[*q,f*]
  End

Command *grant_read_file1*(*p, f, q*)
  If *r* in *a*[*p,f*]
  Then
    Enter *r* into *a*[*q,f*]
  End

Command *grant_read_file2*(*p, f, q*)
  If *c* in *a*[*p,f*]
  Then
    Enter *r* into *a*[*q,f*]
  End

↓

Executing command:
  grant_read_file

is equivalent to executing commands:
  grant_read_file1;
  grant_read_file2

Fundamental questions

- How can we determine that a system is secure?
  - Need to define what we mean by a system being “secure”
- Is there a generic algorithm that allows us to determine whether a computer system is secure?

What is a secure system?

- A simple definition
  - A secure system doesn’t allow violations of a security policy
- Alternative view: based on distribution of rights
  - Leakage of rights: (unsafe with respect to right r)
    - Assume that *A* representing a secure state does not contain a right *r* in an element of A.
    - A right *r* is said to be leaked, if a sequence of operations/commands adds *r* to an element of *A,* which did not contain *r*

What is a secure system?

- Safety of a system with initial protection state X_o
  - Safe with respect to r: System is *safe with respect to r* if *r* can never be leaked

  - Else it is called unsafe with respect to right *r*.

Safety Problem: *formally*

- Given
  - Initial state $X_0 = (S_0, O_0, A_0)$
  - Set of primitive commands c
  - r is not in $A_0[s, o]$
- Can we reach a state X_n where
  - $\exists s,o$ such that $A_n[s,o]$ includes a right r not in $A_0[s,o]$?
    - If so, the system is not safe
    - But is "safe" secure?

Undecidable Problems

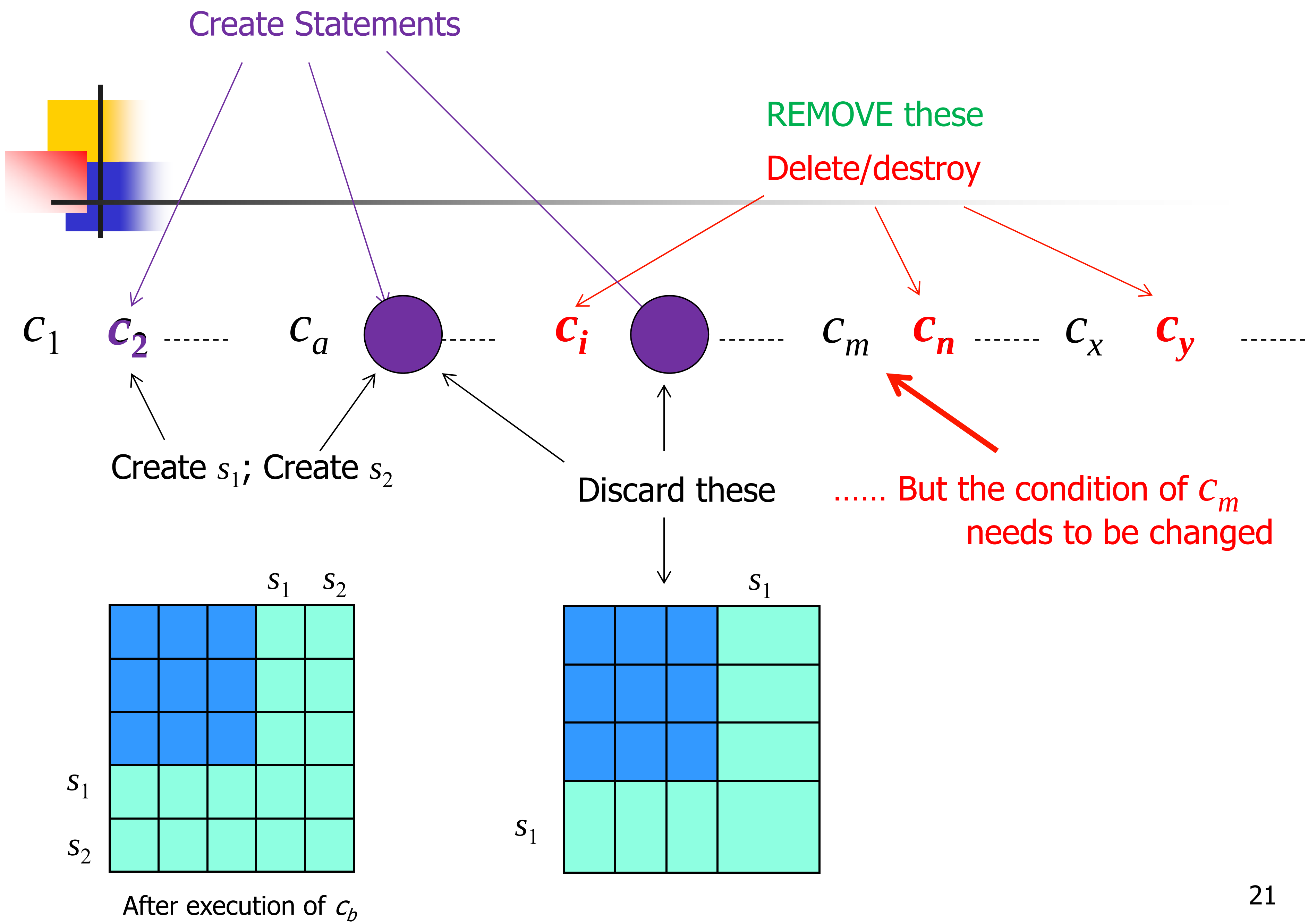
- Decidable Problem
  - A decision problem can be solved by an algorithm that halts on all inputs in a finite number of steps.
- Undecidable Problem
  - A problem that cannot be solved for all cases by any algorithm whatsoever

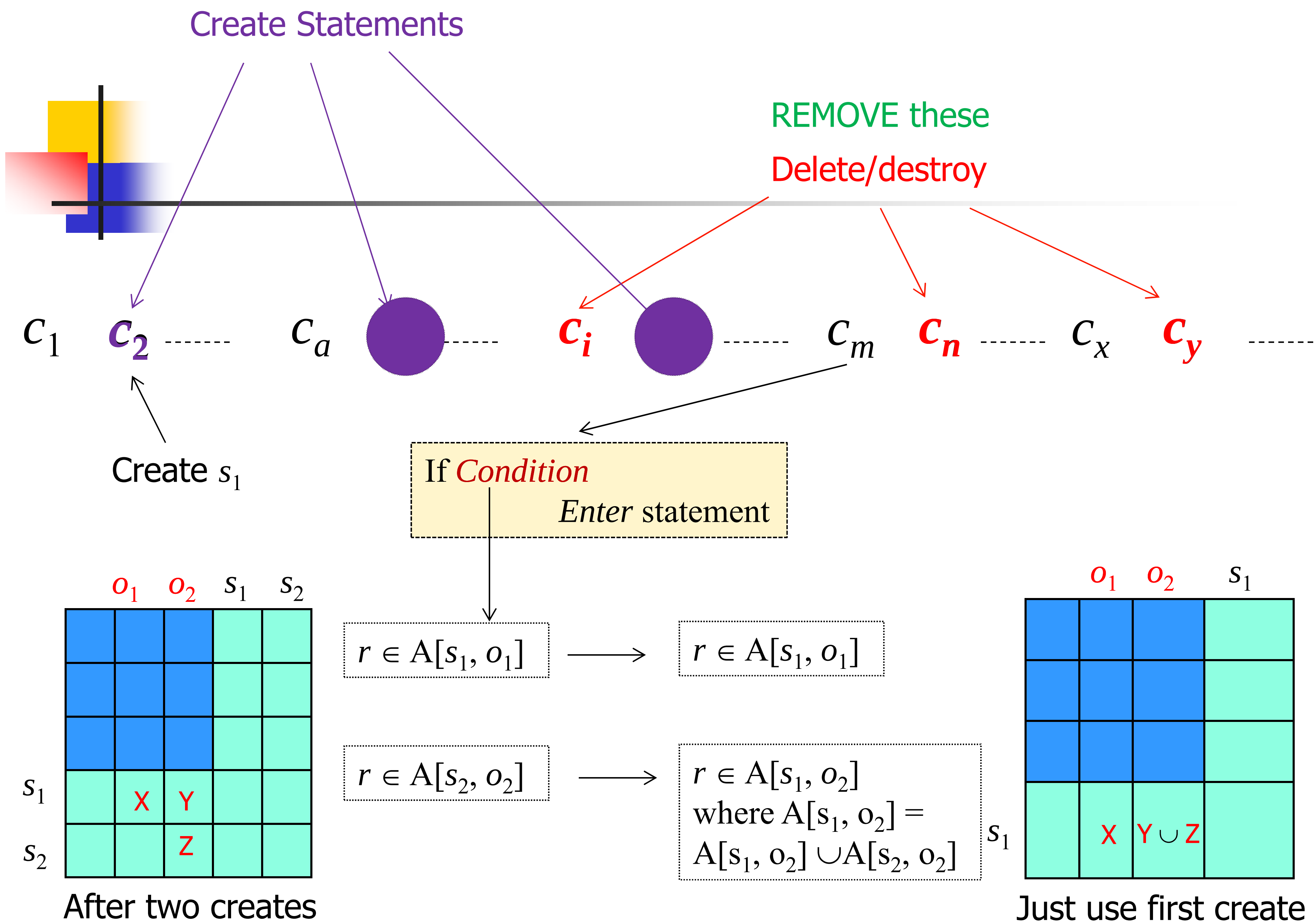
Decidability Results *(Harrison, Ruzzo, Ullman)*

- Theorem:
  - Given a system where each command consists of a single *primitive* command (mono-operational), there exists an algorithm that will determine if a protection system with initial state X_0 is safe with respect to right *r*.

Decidability Results *(Harrison, Ruzzo, Ullman)*

- Proof: determine minimum commands *k* to leak
  - Delete/destroy: Can't leak
  - Create/enter: new subjects/objects "equal", so treat all new subjects as one
    - No test for absence of right
    - Tests on $A[s_1, o_1]$ and $A[s_2, o_2]$ have same result as the same tests on $A[s_1, o_1]$ and $A[s_1, o_2] = A[s_1, o_2] \cup A[s_2, o_2]$
  - If *n* rights leak possible, must be able to leak $k = n(|S_0|+1)(|O_0|+1)+1$ commands
  - Enumerate all possible states to decide


Create Statements
REMOVE these
Delete/destroy
c_1 c_2 ------- c_a ------ c_i ------- c_m c_n ------- c_x c_y -------
Create s_1; Create s_2
Discard these
...... But the condition of c_m needs to be changed
s_1 s_2
s_1
s_2
s_1
s_1
After execution of c_b



Create Statements
REMOVE these
Delete/destroy
c_1 c_2 ------- c_a ------ c_i ------- c_m c_n ------- c_x c_y -------
Create s_1
If *Condition*
Enter statement
o_1 o_2 s_1 s_2
s_1 X Y
s_2 Z
After two creates
$r \in A[s_1, o_1]$
$r \in A[s_1, o_1]$
$r \in A[s_2, o_2]$
$r \in A[s_1, o_2]$
where $A[s_1, o_2] =$
$A[s_1, o_2] \cup A[s_2, o_2]$
o_1 o_2 s_1
s_1 X $Y \cup Z$
Just use first create

Decidability Results *(Harrison, Ruzzo, Ullman)*

- Proof: determine minimum commands k to leak
  - Delete/destroy: Can't leak
  - Create/enter: new subjects/objects "equal", so treat all new subjects as one
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    - Tests on $A[s_1, o_1]$ and $A[s_2, o_2]$ have same result as the same tests on $A[s_1, o_1]$ and $A[s_1, o_2] = A[s_1, o_2] \cup A[s_2, o_2]$
  - If n rights leak possible, must be able to leak $k = n(|S_0|+1)(|O_0|+1)+1$ commands
  - Enumerate all possible states to decide

Decidability Results
(Harrison, Ruzzo, Ullman)

- It is undecidable if a given state of a given protection system is safe for a given generic right
- For proof – need to know Turing machines and halting problem

Turing Machine & halting problem

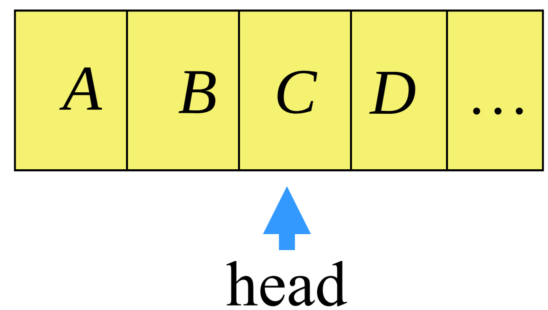
- The **halting problem**:
  - Given a description of an algorithm and a description of its initial arguments, determine whether the algorithm, when executed with these arguments, ever halts (the alternative is that it runs forever without halting).

Turing Machine & Safety problem

- Theorem:
  - It is undecidable if a given state of a given protection system is safe for a given generic right
- Reduce TM to Safety problem
  - If Safety problem is decidable then it implies that TM halts (for all inputs) – showing that the halting problem is decidable (contradiction)
- TM is an abstract model of computer
  - Alan Turing in 1936

Turing Machine

- TM consists of
  - A tape divided into cells; infinite in one direction
  - A set of tape symbols *M*
    - *M* contains a special blank symbol *b*
  - A set of states *K*
  - A head that can read and write symbols
  - An action table that tells the machine how to transition
    - What symbol to write
    - How to move the head ('L' for left and 'R' for right)
    - What is the next state

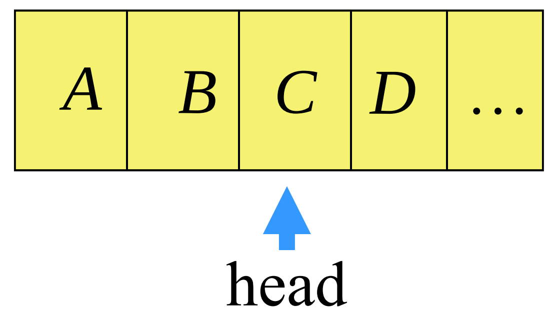


Current state is k

Current symbol is C

Turing Machine

- Transition function $\delta(k, m) = (k', m', \mathrm{L})$:
  - In state k, symbol m on tape location is replaced by symbol m',
  - Head moves one cell to the left, and TM enters state k'
- Halting state is q_f
  - TM halts when it enters this state

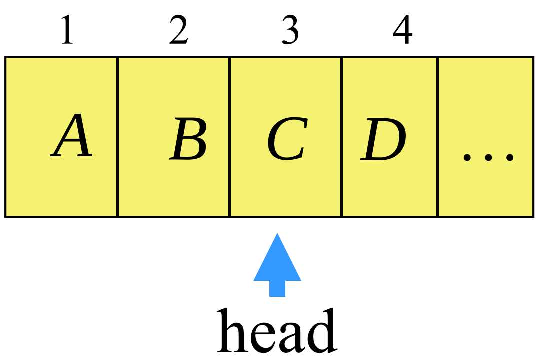


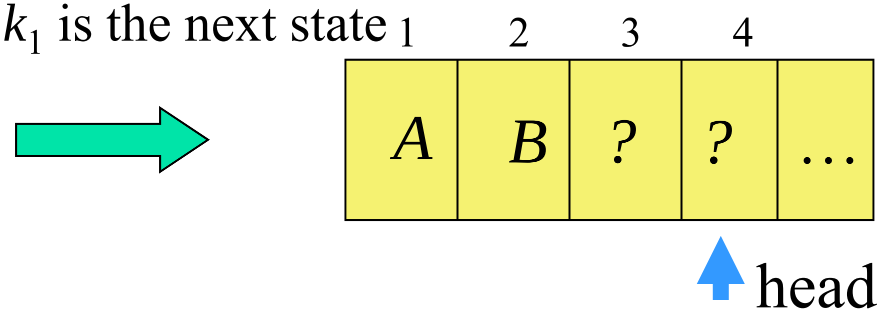
Current state is k
Current symbol is C

Let $\delta(k, C) = (k_1, X, R)$ where k_1 is the next state

Turing Machine

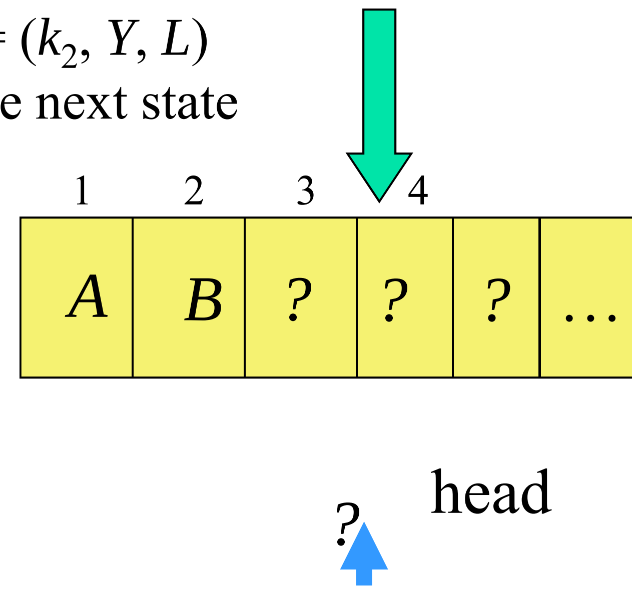
Let $\delta(k, C) = (k_1, X, R)$
where k_1 is the next state



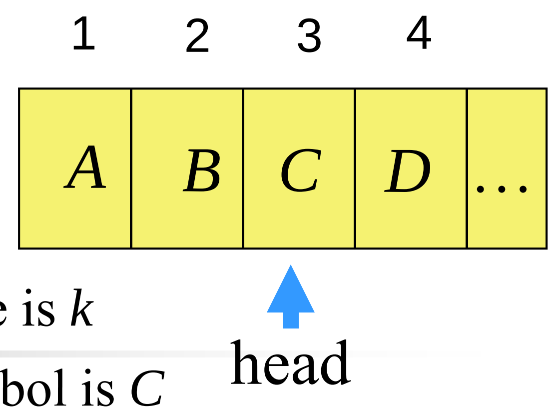


Current state is k
Current symbol is C

Let $\delta(k_1, D) = (k_2, Y, L)$
where k_2 is the next state



TM2Safety Reduction



Current state is k

Current symbol is C

Proof: Reduce TM to safety problem

- Symbols, States $\Rightarrow$ rights
- Tape cell $\Rightarrow$ subject
- Cell s_i has $A \Rightarrow s_i$ has A rights on itself
- Cell $s_k \Rightarrow s_k$ has end rights on itself
- State p, head at $s_i \Rightarrow s_i$ has p rights on itself
- Distinguished Right *own*:
  - s_i *owns* s_{i+1} for $1 \le i < k$

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			C k	*own*	
s_4				D end	

Command Mapping (Left move)

1	2	3	4	
A	B	C	D	…

head

Current state is k

Current symbol is C

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{L})$

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{L})$

If head is not in leftmost

command $c_{k,\mathrm{C}}(s_i, s_{i-1})$
if *own* in $a[s_{i-1}, s_i]$ and k in
    $a[s_i, s_i]$ and C in $a[s_i, s_i]$
then
    delete k from $a[s_i, s_i]$;
    delete C from $a[s_i, s_i]$;
    enter X into $a[s_i, s_i]$;
    enter k_1 into $a[s_{i-1}, s_{i-1}]$;
End

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			C k	*own*	
s_4				D end	

Command Mapping (Left move)

1	2	3	4	
A	B	X	D	…

Current state is k_1

Current symbol is D

head

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{L})$

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{L})$

If head is not in leftmost

command $c_{k,\mathrm{C}}(s_i,\ s_{i-1})$
if own in $a[s_{i-1},\ s_i]$ and k in
  $a[s_i,\ s_i]$ and C in $a[s_i,\ s_i]$
then
  delete k from $a[s_i, s_i]$;
  delete C from $a[s_i, s_i]$;
  enter X into $a[s_i, s_i]$;
  enter k_1 into $a[s_{i-1},\ s_{i-1}]$;
End

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B k_1	*own*		
s_3			X	*own*	
s_4				D end	

If head is in leftmost both s_i and s_{i-1} are s_1

Command Mapping (Right move)

1	2	3	4	
A	*B*	*C*	*D*	…

head

Current state is k

Current symbol is C

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{R})$

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{R})$

command $c_{k,C}(s_i, s_{i+1})$
if *own* in $a[s_i, s_{i+1}]$ and k in
  $a[s_i, s_i]$ and C in $a[s_i, s_i]$
then
  delete k from $a[s_i, s_i]$;
  delete C from $a[s_i, s_i]$;
  enter X into $a[s_i, s_i]$;
  enter k_1 into $a[s_{i+1}, s_{i+1}]$;
end

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			C k	*own*	
s_4				D end	

Command Mapping (Right move)

1	2	3	4	
A	*B*	*X*	*D*	…

head

Current state is k_1

Current symbol is C

$\delta(k, C) = (k_1, X, R)$

$\delta(k, C) = (k_1, X, R)$

command $c_{k,C}(s_i, s_{i+1})$
if *own* in $a[s_i, s_{i+1}]$ and k in
  $a[s_i, s_i]$ and C in $a[s_i, s_i]$
then
  delete k from $a[s_i, s_i]$;
  delete C from $a[s_i, s_i]$;
  enter X into $a[s_i, s_i]$;
  enter k_1 into $a[s_{i+1}, s_{i+1}]$;
end

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				D k_1 end	

Command Mapping (Rightmost move)

1	2	3	4	
A	B	X	D	…

head

Current state is k_1

Current symbol is C

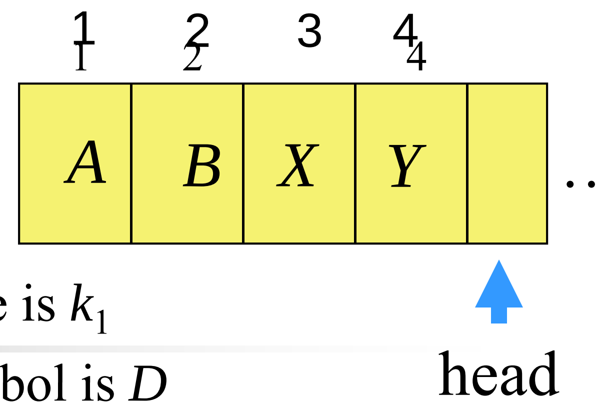
$\delta(k_1, \mathrm{C}) = (k_2, \mathrm{Y}, \mathrm{R})$

$\delta(k_1, \mathrm{D}) = (k_2, \mathrm{Y}, \mathrm{R})$ at end becomes

command $\mathrm{crightmost}_{k,\mathrm{C}}(s_i, s_{i+1})$
if *end* in $a[s_i, s_i]$ and k_1 in $a[s_i, s_i]$
    and D in $a[s_i, s_i]$
then
    delete *end* from $a[s_i, s_i]$;
    create subject s_{i+1};
    enter *own* into $a[s_i, s_{i+1}]$;
    enter *end* into $a[s_{i+1}, s_{i+1}]$;
    delete k_1 from $a[s_i, s_i]$;
    delete D from $a[s_i, s_i]$;
    enter Y into $a[s_i, s_i]$;
    enter k_2 into $a[s_i, s_i]$;
end

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				D k_1 end	

Command Mapping (Rightmost move)



Current state is k_1

Current symbol is D

$\delta(k_1, \mathrm{D}) = (k_2, \mathrm{Y}, \mathrm{R})$

$\delta(k_1, \mathrm{D}) = (k_2, \mathrm{Y}, \mathrm{R})$ at end becomes

command $\mathrm{crightmost}_{k,C}(s_i, s_{i+1})$
if *end* in $a[s_i, s_i]$ and k_1 in $a[s_i, s_i]$
  and D in $a[s_i, s_i]$
then
  delete *end* from $a[s_i, s_i]$;
  create subject s_{i+1};
  enter *own* into $a[s_i, s_{i+1}]$;
  enter *end* into $a[s_{i+1}, s_{i+1}]$;
  delete k_1 from $a[s_i, s_i]$;
  delete D from $a[s_i, s_i]$;
  enter Y into $a[s_i, s_i]$;
  enter k_2 into $a[s_i, s_i]$;
end

	s_1	s_2	s_3	s_4	s_5
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				Y	*own*
s_5					b $k_2$end

Rest of Proof

- Protection system exactly simulates a TM
  - Exactly 1 *end* right in ACM
  - Only 1 right corresponds to a state
  - Thus, at most 1 applicable command in each configuration of the TM
- If TM enters state q_f, then right has leaked
- If safety question decidable, then represent TM as above and determine if q_f leaks
  - Leaks halting state $\Rightarrow$ halting state in the matrix $\Rightarrow$ Halting state reached
- Conclusion: safety question undecidable

Other results

- For protection system without the create primitives, (i.e., delete **create** primitive); the safety question is complete in **P-SPACE**
- It is undecidable whether a given configuration of a given monotonic protection system is safe for a given generic right
  - Delete **destroy**, **delete** primitives;
  - The system becomes monotonic as they only increase in size and complexity
- The safety question for biconditional monotonic protection systems is undecidable
- The safety question for monoconditional, monotonic protection systems is decidable
- The safety question for monoconditional protection systems with **create**, **enter**, **delete** (and no **destroy**) is decidable.

Summary

- HRU Model
- Some foundational results showing that guaranteeing security is hard problem