IS 2150 / TEL 2810
Introduction to Security

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Lecture 4
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Access Control Model
Foundational Results

Objective

- Understand the basic results of the HRU model
  - Saftey issue
  - Turing machine
  - Undecidability

Safety Problem: *formally*

- Given
  - Initial state $X_0 = (S_0, O_0, A_0)$
  - Set of primitive commands c
  - r is not in $A_0[s, o]$
- Can we reach a state X_n where
  - $\exists s,o$ such that $A_n[s,o]$ includes a right r not in $A_0[s,o]$?
    - If so, the system is not safe
    - But is "safe" secure?

Undecidable Problems

- Decidable Problem
  - A decision problem can be solved by an algorithm that halts on all inputs in a finite number of steps.
- Undecidable Problem
  - A problem that cannot be solved for all cases by any algorithm whatsoever

Decidability Results *(Harrison, Ruzzo, Ullman)*

- Theorem:
  - Given a system where each command consists of a single *primitive* command (mono-operational), there exists an algorithm that will determine if a protection system with initial state X_0 is safe with respect to right r.

Decidability Results *(Harrison, Ruzzo, Ullman)*

- Proof: determine minimum commands k to leak
  - Delete/destroy: Can't leak (or be detected)
  - Create/enter: new subjects/objects "equal", so treat all new subjects as one
    - No test for absence
    - Tests on $A[s_1, o_1]$ and $A[s_2, o_2]$ have same result as the same tests on $A[s_1, o_1]$ and $A[s_1, o_2] = A[s_1, o_2] \cup A[s_2, o_2]$
  - If n rights leak possible, must be able to leak $k = n(|S_0|+1)(|O_0|+1)+1$ commands
  - Enumerate all possible states to decide

Decidability Results *(Harrison, Ruzzo, Ullman)*

- It is undecidable if a given state of a given protection system is safe for a given generic right
- For proof – need to know Turing machines and halting problem

Turing Machine & halting problem

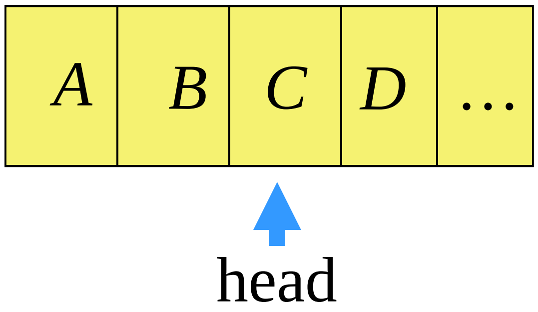
- The **halting problem**:
  - Given a description of an algorithm and a description of its initial arguments, determine whether the algorithm, when executed with these arguments, ever halts (the alternative is that it runs forever without halting).

Turing Machine & Safety problem

- Theorem:
  - It is undecidable if a given state of a given protection system is safe for a given generic right
- Reduce TM to Safety problem
  - If Safety problem is decidable then it implies that TM halts (for all inputs) – showing that the halting problem is decidable (contradiction)
- TM is an abstract model of computer
  - Alan Turing in 1936

Turing Machine

- TM consists of
  - A tape divided into cells; infinite in one direction
  - A set of tape symbols *M*
    - *M* contains a special blank symbol *b*
  - A set of states *K*
  - A head that can read and write symbols
  - An action table that tells the machine how to transition
    - What symbol to write
    - How to move the head ('L' for left and 'R' for right)
    - What is the next state

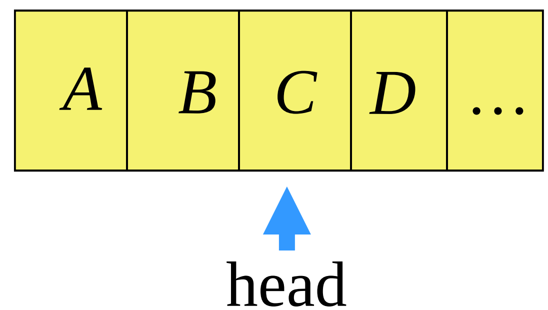


Current state is *k*

Current symbol is *C*

Turing Machine

- Transition function $\delta(k, m) = (k', m', \mathrm{L})$:
  - In state k, symbol m on tape location is replaced by symbol m',
  - Head moves one cell to the left, and TM enters state k'
- Halting state is q_f
  - TM halts when it enters this state

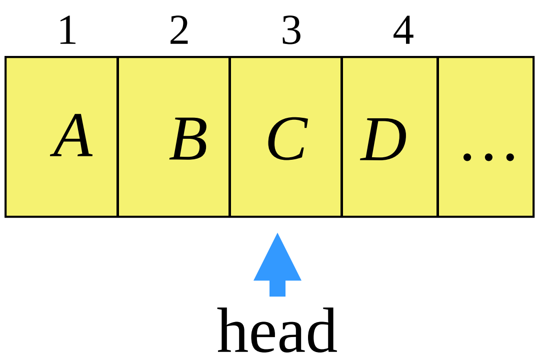


Current state is k

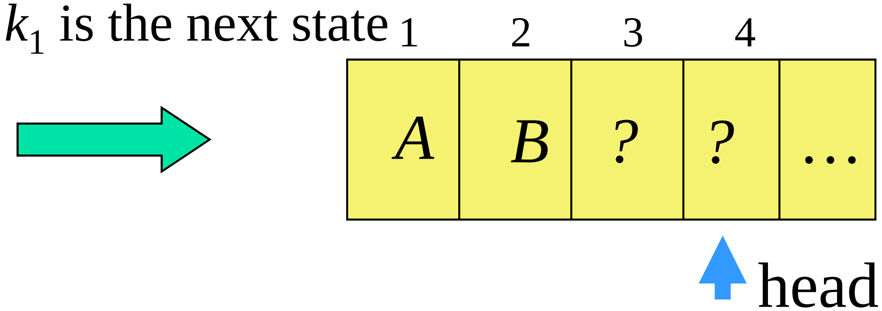
Current symbol is C

Let $\delta(k, C) = (k_1, X, R)$ where k_1 is the next state

Turing Machine

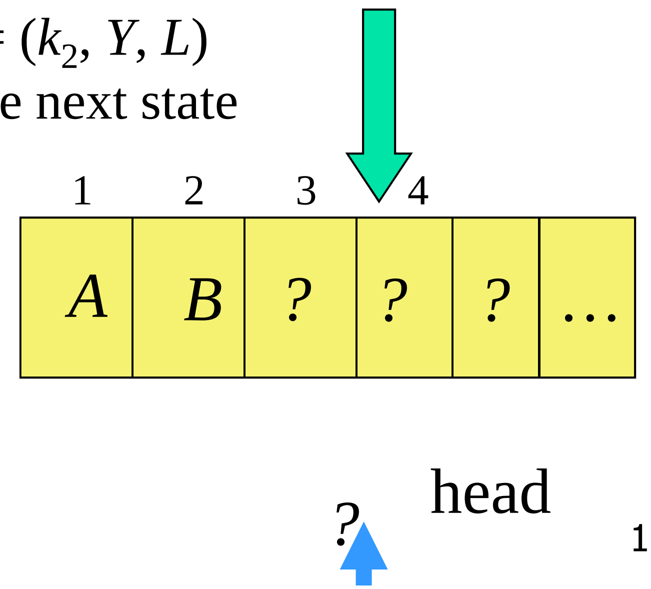


Let $\delta(k, C) = (k_1, X, R)$
where k_1 is the next state

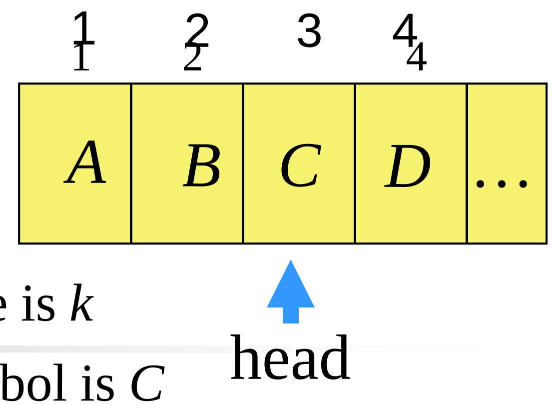


Current state is k
Current symbol is C

Let $\delta(k_1, D) = (k_2, Y, L)$
where k_2 is the next state



TM2Safety Reduction



Current state is k

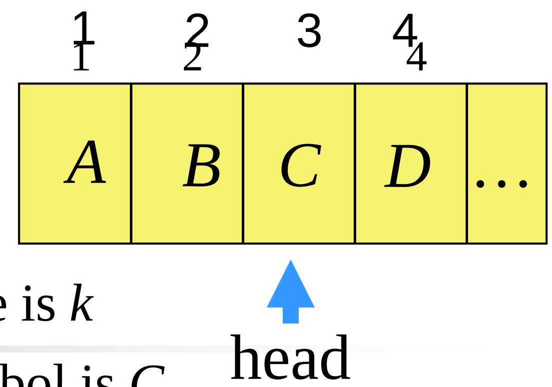
Current symbol is C

Proof: Reduce TM to safety problem

- Symbols, States $\Rightarrow$ rights
- Tape cell $\Rightarrow$ subject
- Cell s_i has $A \Rightarrow s_i$ has A rights on itself
- Cell $s_k \Rightarrow s_k$ has end rights on itself
- State p, head at $s_i \Rightarrow s_i$ has p rights on itself
- Distinguished Right *own*:
  - s_i *owns* s_i*+1* for $1 \le i < k$

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			C k	*own*	
s_4				D end	

Command Mapping (Left move)



Current state is k

Current symbol is C

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{L})$

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{L})$

If head is not in leftmost

command $c_{k,\mathrm{C}}(s_i,\ s_{i-1})$
if *own* in $a[s_{i-1},\ s_i]$ and k in
  $a[s_i,\ s_i]$ and C in $a[s_i,\ s_i]$
then
  delete k from $A[s_i, s_i]$;
  delete C from $A[s_i, s_i]$;
  enter X into $A[s_i, s_i]$;
  enter k_1 into $A[s_{i-1},\ s_{i-1}]$;
End

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			C k	*own*	
s_4				D end	

Command Mapping (Left move)

1	2	3	4	
A	*B*	*X*	*D*	…

head

Current state is k_1

Current symbol is D

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{L})$

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{L})$

If head is not in leftmost

command $c_{k,\mathrm{C}}(s_i,\ s_{i-1})$
if *own* in $a[s_{i-1},\ s_i]$ and k in
  $a[s_i,\ s_i]$ and C in $a[s_i,\ s_i]$
then
  delete k from $A[s_i, s_i]$;
  delete C from $A[s_i, s_i]$;
  enter X into $A[s_i, s_i]$;
  enter k_1 into $A[s_{i-1},\ s_{i-1}]$;
End

If head is in leftmost both s_i and s_{i-1} are s_1

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B k_1	*own*		
s_3			X	*own*	
s_4				D end	

Command Mapping (Right move)

1	2	3	4	
A	B	C	D	…

head

Current state is k

Current symbol is C

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{R})$

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{R})$

command $c_{k,\mathrm{C}}(s_i,\ s_{i+1})$
if *own* in $a[s_i,\ s_{i+1}]$ and k in $a[s_i,\ s_i]$ and C in $a[s_i,\ s_i]$
then
  delete k from $A[s_i, s_i]$;
  delete C from $A[s_i, s_i]$;
  enter X into $A[s_i, s_i]$;
  enter k_1 into $A[s_{i+1}, s_{i+1}]$;
end

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			C k	*own*	
s_4				D end	

Command Mapping (Right move)

1	2	3	4	
A	*B*	*C*	*D*	…

head

Current state is k_1

Current symbol is *C*

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{R})$

$\delta(k, \mathrm{C}) = (k_1, \mathrm{X}, \mathrm{R})$

command $c_{k,\mathrm{C}}(s_i, s_{i+1})$
if *own* in $a[s_i, s_{i+1}]$ and k in $a[s_i, s_i]$ and C in $a[s_i, s_i]$
then
delete k from $A[s_i, s_i]$;
delete C from $A[s_i, s_i]$;
enter X into $A[s_i, s_i]$;
enter k_1 into $A[s_{i+1}, s_{i+1}]$;
end

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				D k_1 end	

Command Mapping (Rightmost move)

1	2	3	4	
A	*B*	*X*	*D*	…

head

Current state is k_1

Current symbol is C

$\delta(k_1, \mathrm{C}) = (k_2, \mathrm{Y}, \mathrm{R})$

$\delta(k_1, \mathrm{D}) = (k_2, \mathrm{Y}, \mathrm{R})$ at end becomes

command $\mathrm{crightmost}_{k,\mathrm{C}}(s_i, s_{i+1})$
if *end* in $a[s_i, s_i]$ and k_1 in $a[s_i, s_i]$
  and D in $a[s_i, s_i]$
then
  delete *end* from $a[s_i, s_i]$;
  create subject s_{i+1};
  enter *own* into $a[s_i, s_{i+1}]$;
  enter *end* into $a[s_{i+1}, s_{i+1}]$;
  delete k_1 from $a[s_i, s_i]$;
  delete D from $a[s_i, s_i]$;
  enter Y into $a[s_i, s_i]$;
  enter k_2 into $A[s_i, s_i]$;
end

	s_1	s_2	s_3	s_4	
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				D k_1 end	

Command Mapping (Rightmost move)

1	2	3	4	
A	*B*	*X*	*D*	…

Current state is k_1

Current symbol is *D*

head

$\delta(k_1, \mathrm{D}) = (k_2, \mathrm{Y}, \mathrm{R})$

$\delta(k_1$, D) = (k_2, Y, R) at end becomes

command crightmost$_{k,C}(s_i, s_{i+1})$
if *end* in $a[s_i, s_i]$ and k_1 in $a[s_i, s_i]$
    and D in $a[s_i, s_i]$
then
    delete *end* from $a[s_i, s_i]$;
    create subject s_{i+1};
    enter *own* into $a[s_i, s_{i+1}]$;
    enter *end* into $a[s_{i+1}, s_{i+1}]$;
    delete k_1 from $a[s_i, s_i]$;
    delete D from $a[s_i, s_i]$;
    enter Y into $a[s_i, s_i]$;
    enter k_2 into $A[s_i, s_i]$;
end

	s_1	s_2	s_3	s_4	s_5
s_1	A	*own*			
s_2		B	*own*		
s_3			X	*own*	
s_4				Y	*own*
s_5					b $k_1$end

Rest of Proof

- Protection system exactly simulates a TM
  - Exactly 1 *end* right in ACM
  - Only 1 right corresponds to a state
  - Thus, at most 1 applicable command in each configuration of the TM
- If TM enters state q_f, then right has leaked
- If safety question decidable, then represent TM as above and determine if q_f leaks
  - Leaks halting state $\Rightarrow$ halting state in the matrix $\Rightarrow$ Halting state reached
- Conclusion: safety question undecidable

Other results

- For protection system without the create primitives, (i.e., delete **create** primitive); the safety question is complete in **P-SPACE**
- It is undecidable whether a given configuration of a given monotonic protection system is safe for a given generic right
  - Delete **destroy**, **delete** primitives;
  - The system becomes monotonic as they only increase in size and complexity
- The safety question for biconditional monotonic protection systems is undecidable
- The safety question for monoconditional, monotonic protection systems is decidable
- The safety question for monoconditional protection systems with **create**, **enter**, **delete** (and no **destroy**) is decidable.