IS 2150 / TEL 2810
Introduction to Security

James Joshi
Associate Professor, SIS

Lecture 7
Oct 20, 2009

Basic Cryptography
Network Security

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Objectives

- Understand/explain/employ the basic cryptographic techniques
  - Review the basic number theory used in cryptosystems
  - Classical system
  - Public-key system
  - Some crypto analysis
  - Message digest

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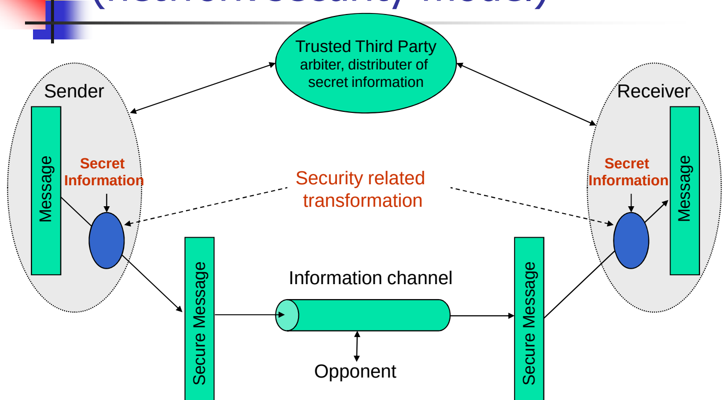
Secure Information Transmission (network security model)



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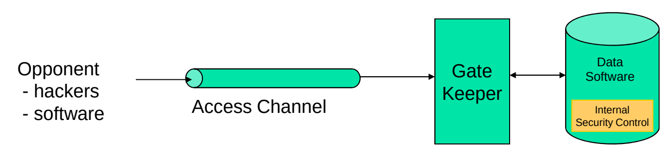
Security of Information Systems (Network access model)



Gatekeeper – firewall or equivalent, password-based login

Internal Security Control – Access control, Logs, audits, virus scans etc.

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Issues in Network security

- Distribution of secret information to enable secure exchange of information
- Effect of communication protocols needs to be considered
- Encryption *if used cleverly and correctly*, can provide several of the security services
- Physical and logical placement of security mechanisms
- Countermeasures need to be considered

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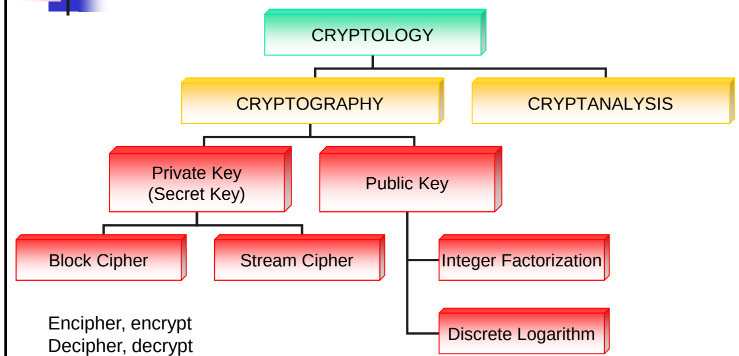
Cryptology



Encipher, encrypt
Decipher, decrypt

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Elementary Number Theory

- Natural numbers N = {1,2,3,…}
- Whole numbers W = {0,1,2,3, …}
- Integers Z = {…,-2,-1,0,1,2,3, …}
- Divisors
  - A number b is said to divide a if $a = mb$ for some m where $a, b, m \in Z$
  - We write this as $b \mid a$

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Divisors

- Some common properties
  - If $a \mid 1$, $a = +1$ or -1
  - If $a \mid b$ and $b \mid a$ then $a = +b$ or $-b$
  - Any $b \in Z$ divides 0 if $b \neq 0$
  - If $b \mid g$ and $b \mid h$ then $b \mid (mg + nh)$ where $b,m,n,g,h \in Z$
- Examples:
  - The positive divisors of 42 are ?
  - $3 \mid 6$ and $3 \mid 21$ => $3 \mid 21m+6n$ for $m,n \in Z$

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Prime Numbers

- An integer p is said to be a prime number if its only positive divisors are 1 and itself
  - 2, 3, 7, 11, ..
- Any integer can be expressed as a ***unique*** product of prime numbers raised to positive integral powers
- Examples
  - 7569 = 3 x 3 x 29 x 29 = 3^2 x 29^2
  - 5886 = 2 x 27 x 109 = 2 x 3^3 x 109
  - 4900 = 7^2 x 5^2 x 2^2
  - 100 = ?
  - 250 = ?
- This process is called ***Prime Factorization***

Greatest common divisor (GCD)

- Definition: Greatest Common Divisor
  - This is the largest divisor of *both a* and *b*
- Given two integers *a* and *b*, the positive integer *c* is called their GCD or greatest common divisor if and only if
  - $c \mid a$ and $c \mid b$
  - Any divisor of both *a* and *b* also divides *c*
- Notation: gcd(*a*, *b*) = *c*
- Example: gcd(49,63) = ?

Relatively Prime Numbers

- Two numbers are said to be relatively prime if their gcd is 1
  - Example: 63 and 22 are relatively prime
- How do you determine if two numbers are relatively prime?
  - Find their GCD or
  - Find their prime factors
    - If they do not have a common prime factor other than 1, they are relatively prime
  - Example: 63 = 9 x 7 = 3^2 x 7 and 22 = 11 x 2

The modulo operation

- What is 27 mod 5?
- Definition
  - Let *a*, *r*, *m* be integers and let $m > 0$
  - We write $a \equiv r \bmod m$ if m divides $r - a$ (or $a - r$) and $0 \leq r < m$
  - *m* is called ?
  - *r* is called ?
  - Note: $a = m.q + r$; *what is q* ?

Modular Arithmetic

- We say that $a \equiv b \bmod m$ if $m \mid a - b$
  - Read as: a is congruent to b modulo m
  - m is called the modulus
  - Example: $27 \equiv 2 \bmod 5$
  - Example: $27 \equiv 7 \bmod 5$ and $7 \equiv 2 \bmod 5$
- $a \equiv b \bmod m \Rightarrow b \equiv a \bmod m$
  - Example: $2 \equiv 27 \bmod 5$
- We usually consider the *smallest positive remainder* which is called the residue

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Modulo Operation

- The modulo operation "reduces" the infinite set of integers to a finite set
- Example: modulo 5 operation
  - We have five sets
    - {...,-10, -5, 0, 5, 10, ...} => $a \equiv 0 \bmod 5$
    - {...,-9,-4,1,6,11,...} => $a \equiv 1 \bmod 5$
    - {...,-8,-3,2,7,12,...} => $a \equiv 2 \bmod 5$, etc.
  - The set of residues of integers modulo 5 has five elements {0,1,2,3,4} and is denoted Z_5.

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Modulo Operation

- Properties
  - $[(a \bmod n) + (b \bmod n)] \bmod n = (a + b) \bmod n$
  - $[(a \bmod n) - (b \bmod n)] \bmod n = (a - b) \bmod n$
  - $[(a \bmod n) \times (b \bmod n)] \bmod n = (a \times b) \bmod n$
  - $(-1) \bmod n = n - 1$
    - (Using $b = q.n + r$, with $b = -1$, $q = -1$ and $r = n-1$)

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Brief History

- All encryption algorithms from BC till 1976 were secret key algorithms
  - Also called private key algorithms or symmetric key algorithms
  - Julius Caesar used a substitution cipher
  - Widespread use in World War II (enigma)
- Public key algorithms were introduced in 1976 by Whitfield Diffie and Martin Hellman

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Cryptosystem

- $(\mathcal{E}, \mathcal{D}, \mathcal{M}, \mathcal{K}, \mathcal{C})$
  - $\mathcal{E}$ set of encryption functions $e: \mathcal{M} \times \mathcal{K} \rightarrow \mathcal{C}$
  - $\mathcal{D}$ set of decryption functions $d: \mathcal{C} \times \mathcal{K} \rightarrow \mathcal{M}$
  - $\mathcal{M}$ set of plaintexts
  - $\mathcal{K}$ set of keys
  - $\mathcal{C}$ set of ciphertexts

Example

- Cæsar cipher
  - $\mathcal{M}$ = { sequences of letters }
  - $\mathcal{K} = \{ i \mid i \text{ is an integer and } 0 \le i \le 25 \}$
  - $\mathcal{E} = \{ E_k \mid k \in \mathcal{K} \text{ and for all letters } m, E_k(m) = (m + k) \bmod 26 \}$
  - $\mathcal{D} = \{ D_k \mid k \in \mathcal{K} \text{ and for all letters } c, D_k(c) = (26 + c - k) \bmod 26 \}$
  - $\mathcal{C} = \mathcal{M}$

Cæsar cipher

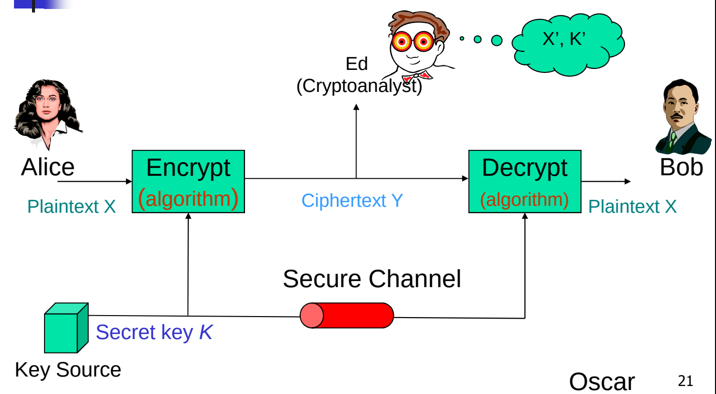
- Let $k = 9$, m = "VELVET" (21 4 11 21 4 19)
  - $E_k(m)$ = (30 13 20 30 13 28) mod 26
    ="4 13 20 4 13 2" = "ENUENC"
  - $D_k(m)$ = (26 + c – k) mod 26
    = (21 30 37 21 30 19) mod 26
    = "21 4 11 21 4 19" = "VELVET"

A	B	C	D	E	F	G	H	I	J	K	L	M
0	1	2	3	4	5	6	7	8	9	10	11	12
N	O	P	Q	R	S	T	U	V	W	X	Y	Z
13	14	15	16	17	18	19	20	21	22	23	24	25

Attacks

- *Ciphertext only*:
  - adversary has only Y;
  - goal ?
- *Known plaintext*:
  - adversary has X, Y;
  - goal ?
- *Chosen plaintext*:
  - adversary gets a specific plaintext enciphered;
  - goal ?

Classical Cryptography



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Classical Cryptography

- Sender, receiver share common key
  - Keys may be the same, or trivial to derive from one another
  - Sometimes called *symmetric cryptography*
- Two basic types
  - Transposition ciphers
  - Substitution ciphers
- Product ciphers
  - Combinations of the two basic types

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Classical Cryptography

- $y = E_k(x)$: Ciphertext → Encryption
- $x = D_k(y)$: Plaintext → Decryption
- k = encryption and decryption key
- The functions $E_k()$ and $D_k()$ must be inverses of one another
  - $E_k(D_k(y)) = ?$
  - $D_k(E_k(x)) = ?$
  - $E_k(D_k(x)) = ?$

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Transposition Cipher

- Rearrange letters in plaintext to produce ciphertext
- Example (Rail-Fence Cipher)
  - Plaintext is "`HELLO WORLD`"
  - Rearrange as

    `HLOOL`

    `ELWRD`
  - Ciphertext is `HLOOL ELWRD`

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Attacking the Cipher

- Anagramming
  - If 1-gram frequencies match English frequencies, but other *n*-gram frequencies do not, probably transposition
  - Rearrange letters to form *n*-grams with highest frequencies

Example

- Ciphertext: `HLOOLELWRD`
- Frequencies of 2-grams beginning with H
  - HE 0.0305
  - HO 0.0043
  - HL, HW, HR, HD < 0.0010
- Frequencies of 2-grams ending in H
  - WH 0.0026
  - EH, LH, OH, RH, DH ≤ 0.0002
- Implies E follows H

Example

- Arrange so that H and E are adjacent

```
HE
LL
OW
OR
LD
```

- Read off across, then down, to get original plaintext

Substitution Ciphers

- Change characters in plaintext to produce ciphertext
- Example (Cæsar cipher)
  - Plaintext is `HELLO WORLD`;
  - Key is 3, usually written as letter 'D'
  - Ciphertext is `KHOOR ZRUOG`

Attacking the Cipher

- Brute Force: Exhaustive search
  - If the key space is small enough, try all possible keys until you find the right one
  - Cæsar cipher has 26 possible keys
- Statistical analysis
  - Compare to 1-gram model of English

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Statistical Attack

- Ciphertext is `KHOOR ZRUOG`
- Compute frequency of each letter in ciphertext:

  G 0.1 H 0.1 K 0.1 O 0.3

  R 0.2 U 0.1 Z 0.1
- Apply 1-gram model of English
  - Frequency of characters (1-grams) in English is on next slide

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Character Frequencies (Denning)

a	0.080	h	0.060	n	0.070	t	0.090
b	0.015	i	0.065	o	0.080	u	0.030
c	0.030	j	0.005	p	0.020	v	0.010
d	0.040	k	0.005	q	0.002	w	0.015
e	0.130	l	0.035	r	0.065	x	0.005
f	0.020	m	0.030	s	0.060	y	0.020
g	0.015					z	0.002

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Statistical Analysis

- $f(c)$ frequency of character c in ciphertext
- $\varphi(i)$:
  - correlation of frequency of letters in ciphertext with corresponding letters in English, assuming key is i
  - $\varphi(i) = \Sigma_{0 \le c \le 25} f(c)p(c - i)$
  - so here,
    $\varphi(i) = 0.1p(6 - i) + 0.1p(7 - i) + 0.1p(10 - i) + 0.3p(14 - i) + 0.2p(17 - i) + 0.1p(20 - i) + 0.1p(25 - i)$
    - $p(x)$ is frequency of character x in English
  - Look for maximum correlation!

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Correlation: $\varphi(i)$ for $0 \le i \le 25$

i	$\varphi(i)$	i	$\varphi(i)$	i	$\varphi(i)$	i	$\varphi(i)$
0	0.0482	7	0.0442	13	0.0520	19	0.0315
1	0.0364	8	0.0202	14	0.0535	20	0.0302
2	0.0410	9	0.0267	15	0.0226	21	0.0517
3	0.0575	10	0.0635	16	0.0322	22	0.0380
4	0.0252	11	0.0262	17	0.0392	23	0.0370
5	0.0190	12	0.0325	18	0.0299	24	0.0316
6	0.0660					25	0.0430

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The Result

- Ciphertext is `KHOOR ZRUOG`
- Most probable keys, based on φ:
  - $i = 6$, $\varphi(i) = 0.0660$
    - plaintext `EBIIL TLOLA` (How?)
  - $i = 10$, $\varphi(i) = 0.0635$
    - plaintext `AXEEH PHKEW` (How?)
  - $i = 3$, $\varphi(i) = 0.0575$
    - plaintext `HELLO WORLD` (How?)
  - $i = 14$, $\varphi(i) = 0.0535$
    - plaintext `WTAAD LDGAS`
- Only English phrase is for $i = 3$
  - That's the key (3 or 'D')

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Cæsar's Problem

- Key is too short
  - Can be found by exhaustive search
  - Statistical frequencies not concealed well
    - They look too much like regular English letters
- So make it longer
  - Multiple letters in key
  - Idea is to smooth the statistical frequencies to make cryptanalysis harder

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Vigenère Cipher

- Like Cæsar cipher, but use a phrase
- Example
  - Message `THE BOY HAS THE BALL`
  - Key `VIG`
  - Encipher using Cæsar cipher for each letter:

```
key    VIGVIGVIGVIGVIGV
plain  THEBOYHASTHEBALL
cipher OPKWWECIYOPKWIRG
```

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Relevant Parts of Tableau

	G	*I*	*V*
A	G	I	V
B	H	J	W
E	K	M	Z
H	N	P	C
L	R	T	G
O	U	W	J
S	Y	A	N
T	Z	B	O
Y	E	H	T

- Tableau with relevant rows, columns only
- Example encipherments:
  - key V, letter T: follow V column down to T row (giving "O")
  - Key I, letter H: follow I column down to H row (giving "P")

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Useful Terms

- *period*: length of key
  - In earlier example, period is 3
- *tableau*: table used to encipher and decipher
  - Vigènere cipher has key letters on top, plaintext letters on the left
- *polyalphabetic*: the key has several different letters
  - Cæsar cipher is monoalphabetic

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Attacking the Cipher

- Key to attacking vigenère cipher
  - determine the key length
  - If the keyword is n, then the cipher consists of n monoalphabetic substitution ciphers

key	VIGVIGVIGVIGVIGV
plain	THEBOYHASTHEBALL
cipher	OPKWWECIYOPKWIRG

key	DECEPTIVEDECEPTIVEDECEPTIVE
plain	WEAREDISCOVEREDSAVEYOURSELF
cipher	ZICVTWQNGRZGVTWAVZHCQYGLMGJ

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One-Time Pad

- A Vigenère cipher with a random key at least as long as the message
  - Provably unbreakable; Why?
  - Consider ciphertext DXQR. Equally likely to correspond to
    - plaintext DOIT (key AJIY) and
    - plaintext DONT (key AJDY) and any other 4 letters
  - Warning: keys *must* be random, or you can attack the cipher by trying to regenerate the key

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Overview of the DES

- A block cipher:
  - encrypts blocks of 64 bits using a 64 bit key
  - outputs 64 bits of ciphertext
  - A product cipher
    - performs both substitution and transposition (permutation) on the bits
  - basic unit is the bit
- Cipher consists of 16 rounds (iterations) each with a round key generated from the user-supplied key

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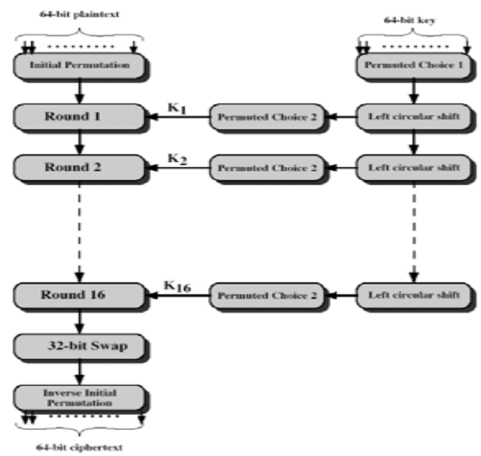
DES



- Round keys are 48 bits each
  - Extracted from 64 bits
  - Permutation applied
- Deciphering involves using round keys in reverse

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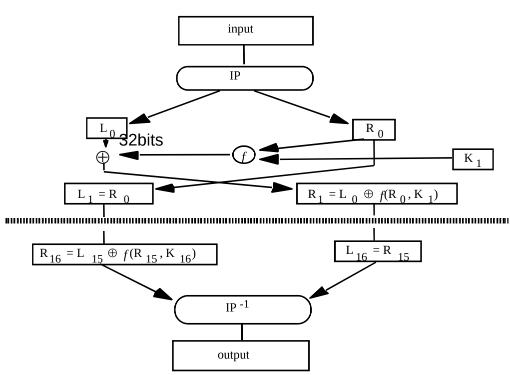
Encipherment



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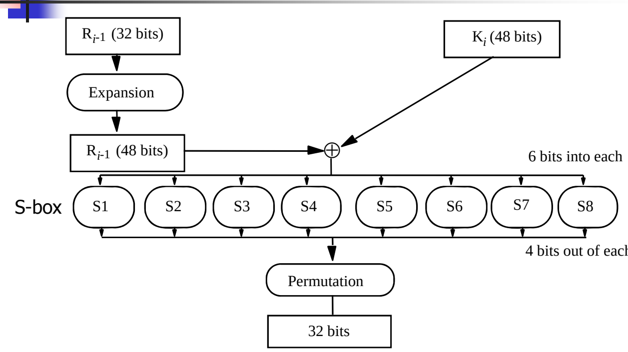
The *f* Function



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Controversy

- Considered too weak
  - Design to break it using 1999 technology published
  - Design decisions not public
    - S-boxes may have backdoors
- Several other weaknesses found
  - Mainly related to keys

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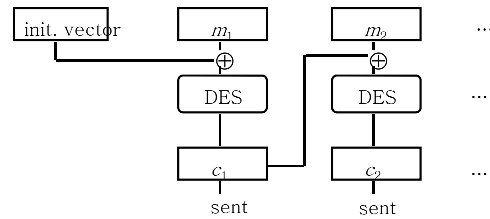
DES Modes

- Electronic Code Book Mode (ECB):
  - Encipher each block independently
- Cipher Block Chaining Mode (CBC)
  - XOR each block with previous ciphertext block
  - Uses an initialization vector for the first one



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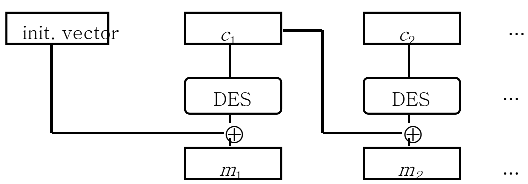
CBC Mode Decryption



- CBC has self healing property
  - If one block of ciphertext is altered, the error propagates for at most two blocks

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Self-Healing Property

- Initial message
  - 3231343336353837 3231343336353837 3231343336353837 3231343336353837
- Received as (underlined 4c should be 4b)
  - ef7c4cb2b4ce6f3b f6266e3a97af0e2c 746ab9a6308f4256 33e60b451b09603d
- Which decrypts to
  - efca61e19f4836f1 3231333336353837 3231343336353837 3231343336353837
  - Incorrect bytes underlined; plaintext “heals” after 2 blocks

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Public Key Cryptography

- Two keys
  - *Private key* known only to individual
  - *Public key* available to anyone
- Idea
  - Confidentiality:
    - encipher using public key,
    - decipher using private key
  - Integrity/authentication:
    - encipher using private key,
    - decipher using public one

Requirements

1. Given the appropriate key, it must be computationally easy to encipher or decipher a message
2. It must be computationally infeasible to derive the private key from the public key
3. It must be computationally infeasible to determine the private key from a chosen plaintext attack

Diffie-Hellman

- Compute a common, shared key
  - Called a *symmetric key exchange protocol*
- Based on discrete logarithm problem
  - Given integers n and g and prime number p, compute k such that $n = g^k \bmod p$
  - Solutions known for small p
  - Solutions computationally infeasible as p grows large – hence, choose large p

Algorithm

- Constants known to participants
  - prime p; integer g other than 0, 1 or p–1
- Alice: (private = k_A, public = K_A)
- Bob: (private = k_B, public = K_B)
  - $K_A = g^{kA} \bmod p$
  - $K_B = g^{kB} \bmod p$
- To communicate with Bob,
  - Alice computes $S_{A, B} = K_B^{kA} \bmod p$
- To communicate with Alice,
  - Bob computes $S_{B, A} = K_A^{kB} \bmod p$
- $S_{A, B} = S_{B, A}$?

Example

- Assume $p = 53$ and $g = 17$
- Alice chooses $k_A = 5$
  - Then $K_A = 17^5 \bmod 53 = 40$
- Bob chooses $k_B = 7$
  - Then $K_B = 17^7 \bmod 53 = 6$
- Shared key:
  - $K_B^{kA} \bmod p = 6^5 \bmod 53 = 38$
  - $K_A^{kB} \bmod p = 40^7 \bmod 53 = 38$

Exercise:

Let p = 5, g = 3
$k_A = 4$, $k_B = 3$

$K_A = ?$, $K_B = ?$,
$S = ?$,

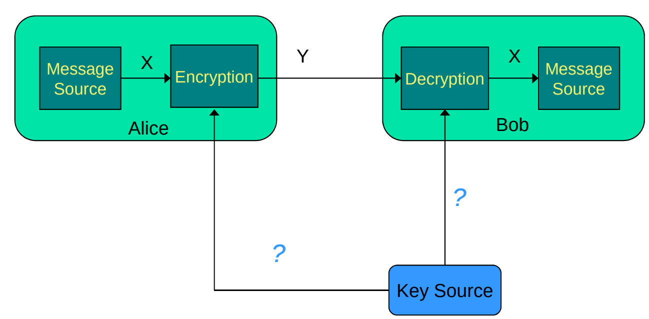
RSA

- Relies on the difficulty of determining the number of numbers relatively prime to a large integer n
- Totient function $\phi(n)$
  - Number of + integers less than n and relatively prime to n
- Example: $\phi(10) = 4$
  - What are the numbers relatively prime to 10?
- $\phi(77)$?
- $\phi(p)$? When p is a prime number
- $\phi(pq)$? When p and q are prime numbers

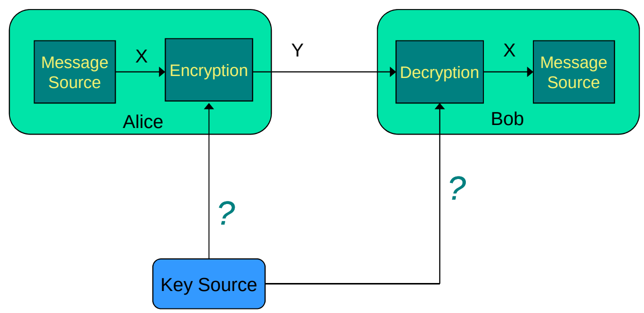
Algorithm

- Choose two large prime numbers p, q
  - Let $n = pq$, then $\phi(n) = (p-1)(q-1)$
  - Choose $e < n$ relatively prime to $\phi(n)$.
  - Compute d such that $ed \bmod \phi(n) = 1$
    - Public key: (e, n);
    - private key: d (*or* (d, n))
- Encipher: $c = m^e \bmod n$
- Decipher: $m = c^d \bmod n$

Confidentiality using RSA



Authentication using RSA



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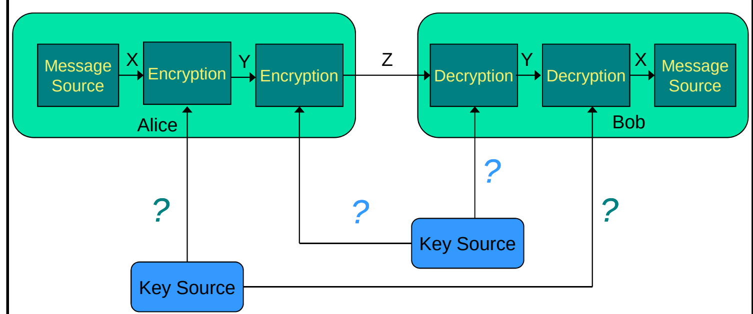
Confidentiality + Authentication



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Warnings

- Encipher message in blocks considerably larger than the examples here
  - If 1 character per block, RSA can be broken using statistical attacks (just like classical cryptosystems)
  - Attacker cannot alter letters, but can rearrange them and alter message meaning
    - Example: reverse enciphered message: ON to get NO

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Cryptographic Checksums

- Mathematical function to generate a set of k bits from a set of n bits (where $k \leq n$).
  - k is smaller then n except in unusual circumstances
  - Keyed CC: requires a cryptographic key
    $h = C_{Key}(M)$
  - Keyless CC: requires no cryptographic key
    - Message Digest or One-way Hash Functions
    $h = H(M)$
- Can be used for message authentication
  - Hence, also called Message Authentication Code (MAC)

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Mathematical characteristics

- Every bit of the message digest function potentially influenced by every bit of the function's input
- If any given bit of the function's input is changed, every output bit has a 50 percent chance of changing
- Given an input file and its corresponding message digest, it should be computationally infeasible to find another file with the same message digest value

Definition

- Cryptographic checksum function $h: A \to B$:
  1. For any $x \in A$, $h(x)$ is easy to compute
     - Makes hardware/software implementation easy
  2. For any $y \in B$, it is computationally infeasible to find $x \in A$ such that $h(x) = y$
     - *One-way property*
  3. It is computationally infeasible to find $x, x' \in A$ such that $x \neq x'$ and $h(x) = h(x')$
  4. Alternate form: Given any $x \in A$, it is computationally infeasible to find a different $x' \in A$ such that $h(x) = h(x')$.

Collisions

- If $x \neq x'$ and $h(x) = h(x')$, x and x' are a collision
  - Pigeonhole principle: if there are n containers for $n+1$ objects, then at least one container will have 2 objects in it.
  - Application: suppose $n = 5$ and $k = 3$. Then there are 32 elements of A and 8 elements of B, so
    - each element of B has at least 4 corresponding elements of A

Keys

- Keyed cryptographic checksum: requires cryptographic key
  - DES in chaining mode: encipher message, use last n bits. Requires a key to encipher, so it is a keyed cryptographic checksum.
- Keyless cryptographic checksum: requires no cryptographic key
  - MD5 and SHA-1 are best known; others include MD4, HAVAL, and Snefru

Message Digest

- MD2, MD4, MD5 (Ronald Rivest)
  - Produces 128-bit digest;
  - MD2 is probably the most secure, longest to compute (hence rarely used)
  - MD4 is a fast alternative; MD5 is modification of MD4
- SHA, SHA-1 (Secure Hash Algorithm)
  - Related to MD4; used by NIST's Digital Signature
  - Produces 160-bit digest
  - SHA-1 may be better
- SHA-256, SHA-384, SHA-512
  - 256-, 384-, 512 hash functions designed to be use with the Advanced Encryption Standards (AES)
- Example:
  - MD5(There is $1500 in the blue bo) = f80b3fde8ecbac1b515960b9058de7a1
  - MD5(There is $1500 in the blue box) = a4a5471a0e019a4a502134d38fb64729

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Hash Message Authentication Code (HMAC)

- Make keyed cryptographic checksums from keyless cryptographic checksums
- h be keyless cryptographic checksum function that takes data in blocks of b bytes and outputs blocks of l bytes. k' is cryptographic key of length b bytes (from k)
  - If short, pad with 0s' to make b bytes; if long, hash to length b
- $ipad$ is 00110110 repeated b times
- $opad$ is 01011100 repeated b times
- HMAC-$h(k, m) = h(k' \oplus opad \,||\, h(k' \oplus ipad \,||\, m))$
  - $\oplus$ exclusive or, $||$ concatenation

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Protection Strength

- Unconditionally Secure
  - Unlimited resources + unlimited time
  - Still the plaintext CANNOT be recovered from the ciphertext
- Computationally Secure
  - Cost of breaking a ciphertext exceeds the value of the hidden information
  - The time taken to break the ciphertext exceeds the useful lifetime of the information

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Average time required for exhaustive key search

Key Size (bits)	Number of Alternative Keys	Time required at 10^6 Decryption/µs
32	$2^{32} = 4.3 \times 10^9$	2.15 milliseconds
56	$2^{56} = 7.2 \times 10^{16}$	10 hours
128	$2^{128} = 3.4 \times 10^{38}$	5.4×10^{18} years
168	$2^{168} = 3.7 \times 10^{50}$	5.9×10^{30} years

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Key Points

- Two main types of cryptosystems: classical and public key
- Classical cryptosystems encipher and decipher using the same key
  - Or one key is easily derived from the other
- Public key cryptosystems encipher and decipher using different keys
  - Computationally infeasible to derive one from the other

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