IS 2610: Data Structures

Searching

March 29, 2004

Symbol Table

- A symbol table is a data structure of items with keys that supports two basic operations: *insert* a new item, and *return* an item with a given key
  - Examples:
    - Account information in banks
    - Airline reservations

Symbol Table ADT

- Key operations
  - Insert a new item
  - Search for an item with a given key
  - Delete a specified item
  - Select the k^{th} smallest item
  - Sort the symbol table
  - Join two symbol tables

```
void STinit(int);
 int STcount();
void STinsert(Item);
Item STsearch(Key);
void STdelete(Item);
Item STselect(int);
void STsort(void (*visit)(Item));
```

Key-indexed ST

- Simplest search algorithm is based on storing items in an array, indexed by the keys

```
static Item *st;
static int M = maxKey;
void STinit(int maxN)
  { int i;
    st = malloc((M+1)*sizeof(Item));
    for (i = 0; i <= M; i++) st[i] = NULLitem;
  }
```

```
int STcount()
  { int i, N = 0;
    for (i = 0; i < M; i++)
      if (st[i] != NULLitem) N++;
    return N;
  }
void STinsert(Item item)
  { st[key(item)] = item; }
Item STsearch(Key v)
  { return st[v]; }
void STdelete(Item item)
  { st[key(item)] = NULLitem; }
Item STselect(int k)
  { int i;
    for (i = 0; i < M; i++)
      if (st[i] != NULLitem)
        if (k-- == 0) return st[i];
  }
void STsort(void (*visit)(Item))
  { int i;
    for (i = 0; i < M; i++)
      if (st[i] != NULLitem) visit(st[i]);
  }
```

Sequential Search based ST

- When a new item is inserted, we put it into the array by moving the larger elements over one position (as in insertion sort)
- To search for an element
  - Look through the array sequentially
  - If we encounter a key larger than the search key – we report an error

Binary Search

- Divide and conquer methodology
  - Divide the items into two parts
  - Determine which part the search key belongs to and concentrate on that part
    - Keep the items sorted
    - Use the indices to delimit the part searched.

```
Item search(int l, int r, Key v)
  { int m = (l+r)/2;
    if (l > r) return NULLitem;
    if eq(v, key(st[m])) return st[m];
    if (l == r) return NULLitem;
    if less(v, key(st[m]))
         return search(l, m-1, v);
    else return search(m+1, r, v);
  }
Item STsearch(Key v)
  { return search(0, N-1, v); }
```

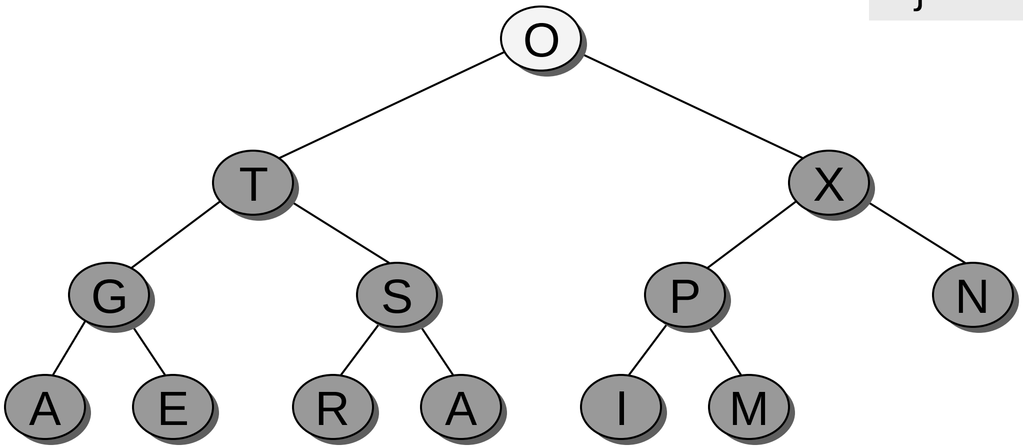
Binary Search Tree

- NST is a binary tree
  - A key is associated with each of its internal nodes
  - Key in any node
    - is larger than (or equal to) the keys in all nodes in that node's left subtree
    - is smaller than (or equal to) the keys in all nodes in that node's right subtree
- What is the output of inorder traversal on BST?

BST insertion

- Insert L !!

```
void STinsert(Item item)
  { Key v = key(item); link p = head, x = p;
    if (head == NULL)
      { head = NEW(item, NULL, NULL, 1); return; }
    while (x != NULL)
      {
        p = x; x->N++;
        x = less(v, key(x->item)) ? x->l : x->r;
      }
    x = NEW(item, NULL, NULL, 1);
    if (less(v, key(p->item))) p->l = x;
                               else p->r = x;
  }
```




```
link insertR(link h, Item item)
  { Key v = key(item), t = key(h->item);
    if (h == z) return NEW(item, z, z, 1);
    if less(v, t)
         h->l = insertR(h->l, item);
    else h->r = insertR(h->r, item);
    (h->N)++; return h;
  }
void STinsert(Item item)
  { head = insertR(head, item); }
```

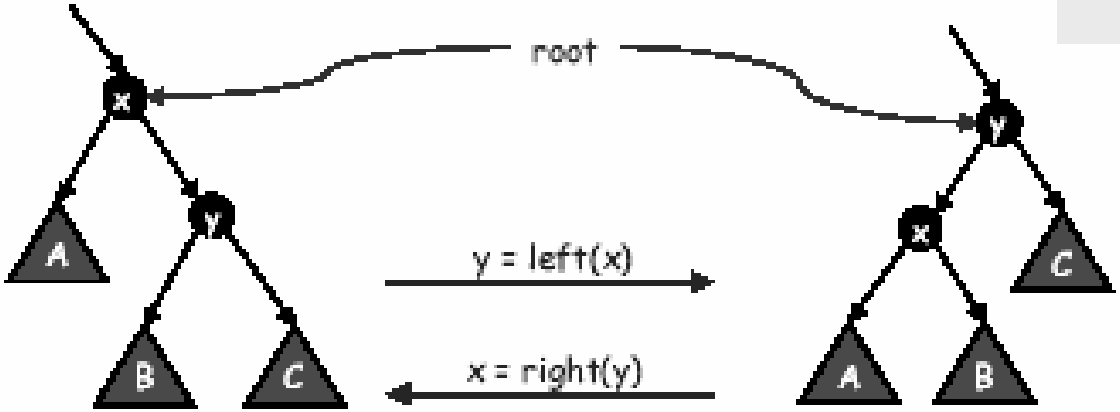
BST Complexities

- Best and worst case heights
  - $ln\ N$ and N
- Search costs
  - Internal path length is related to – search hit
  - External path length is related to – search miss
- N random keys
  - Average: Insertion, Search hit and Search miss require about $2\ ln\ N$ comparisons
  - Worst case search: N comparisons

Basic Rotations

- Transformations to rearrange nodes in a tree
  - Maintain BST
  - Changes three pointers

```
link rotL(link h)
  { link x = h->r; h->r = x->l; x->l = h;
    return x; }
link rotR(link h)
  { link x = h->l; h->l = x->r; x->r = h;
    return x; }
```

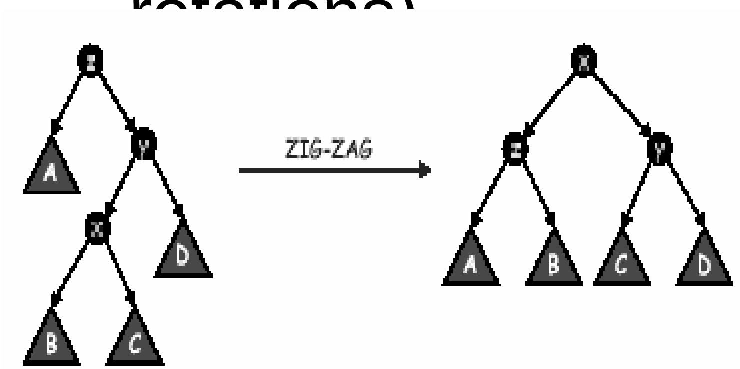


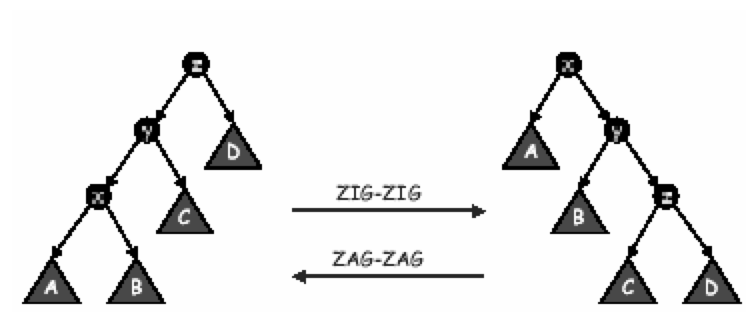
Balanced Trees

- BST – worst case is bad!!
- Keep trees balanced so that searches can be done in less than $ln\ N + 1$ comparisons
  - Maintenance cost incurred!
- Splay trees (Self-adjusting)
  - Tree automatically reorganizes itself after each op
  - When insert or search for x, rotate x up to root using “double rotations”
  - Tree remains “balanced” without explicitly storing any balance information

Splay trees

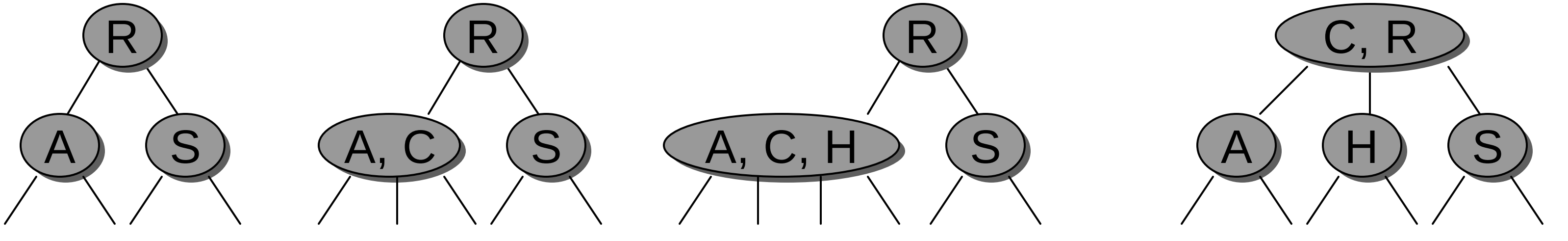
- Check two links above current node
  - ZIG-ZAG: if orientations differ, same as root insertion
  - ZIG-ZIG: if orientations match, do top rotation first (unlike bottom rotation in root insertion using basic rotations)





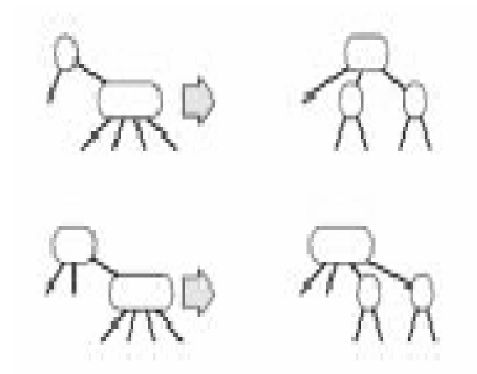
2-3-4 Trees

- Nodes can hold more than one key
  - 2-nodes : 1 key; two links
  - 3-nodes : 2 keys; three links
  - 4-nodes : 3 keys; four links
- A balanced 2-3-4 tree
  - Links to empty trees are at the same hieght

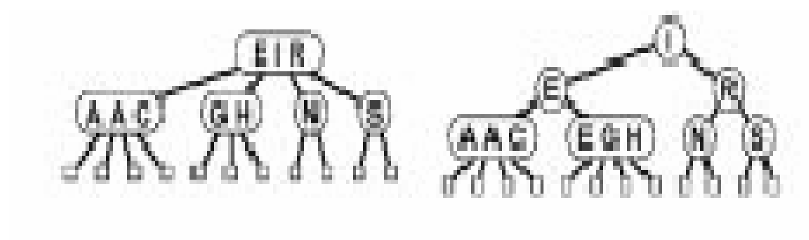


2-3-4 Trees

- How doe you Search?
- Insert
  - Search to bottom for key
  - 2-node at bottom: convert to 3-node
  - 3-node at bottom: convert to 4-node
  - 4-node at bottom – *split*
- Whenever root becomes 4 node – split it into a triangle of three 2-nodes

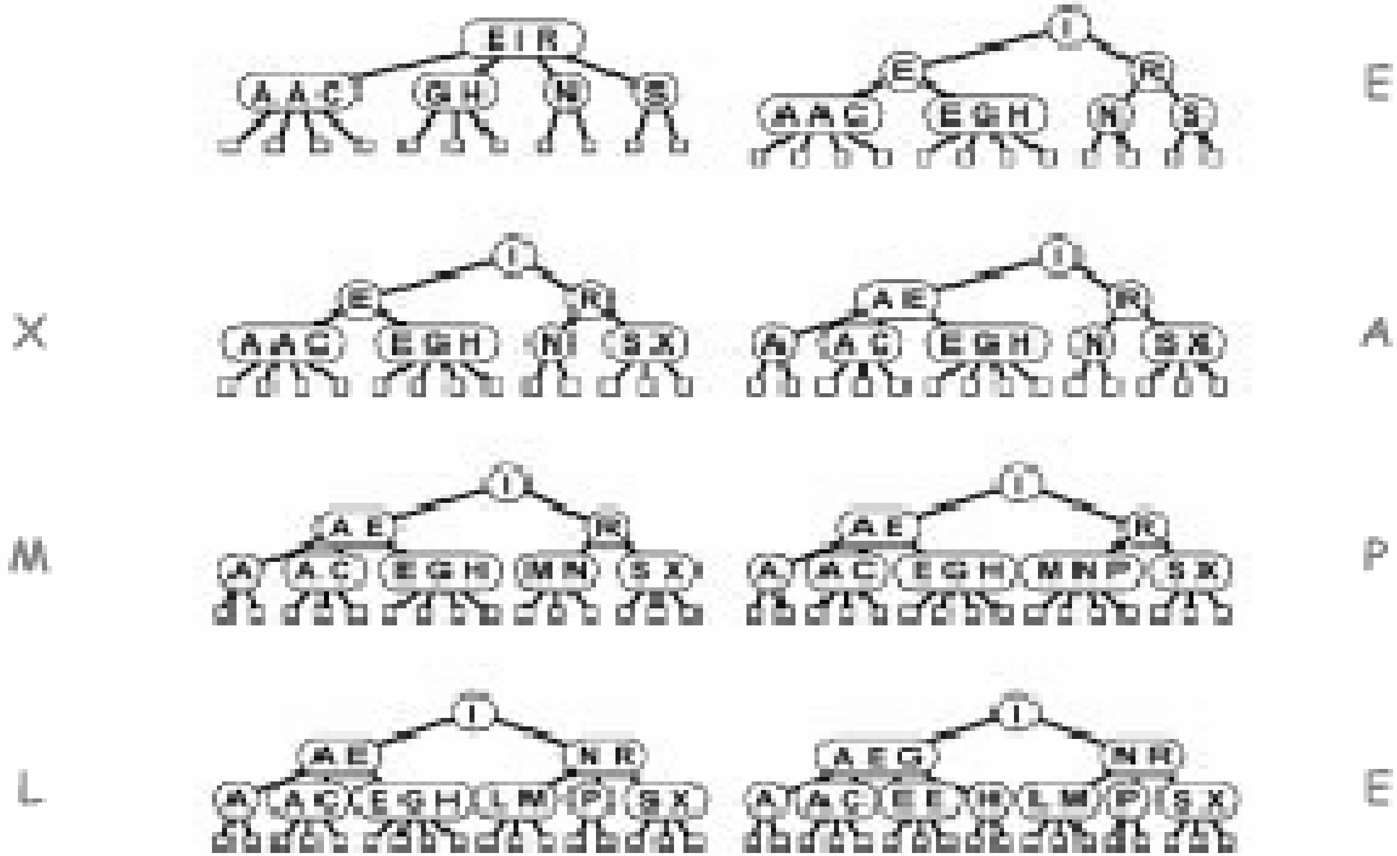


Add E



Red black trees

- Represent 2-3-4 trees as binary trees


E
X
A
M
P
L
E

Hashing

- Save items in a key-indexed table
  - Index is a function of the key
- Hash function
  - function to compute table index from search key
- Collision resolution strategy
  - Algorithms and data structures to handle two keys that hash to the same index
  - One approach – use linked list

Hashing

- Time-space complexity
  - No space limitation
    - Any search can be done in one memory access
  - No time limitation
    - Use limited memory and do sequential search
  - Limitation on both
    - Hashing to balance

Hash function: h

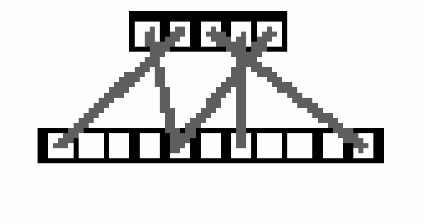
- Given a hash table of size M
  - h(Key) is a value in [0,.., M]
  - Ideally, for each input, every output should be equally likely
- Simple methods
  - Modular hash function
    - h(K) = K mod M; choose M as prime
  - Multiplicative and modular methods
    - h(K) = (Kα) mod M; choose M as prime
    - A popular choice is α= 0.618033 (golden ration)

Hash Function: h

- Strings of characters
  - $26^4 \approx .5$ Million 4-char keys
  - Table size M = 101

Binary	01100001	01100010	01100011	01100100
Hex	61	62	63	64
Dec	97	98	99	100
ascii	a	b	c	d

- abcd hashes to 11
  - 0x61626364 % 101 = 16338831724 %



- dcba hashes to 57
  - Collision is inevitable

Hash function: h

- Horner’s method
  - 0x61626364 = 256*(256*(256*97+98) + 99)+100
  - 0x61626364 mod 101 = 256*(256*(256*97+98) + 99)+100 mod 101
  - Can take mod after each op
    - (256*97+98) mod 101 = 84
    - (256*84+99) mod 101 = 90
    - (256*90+100) mod 101 = 11
  - N add, multiply and mod ops

```
int hash(char *v, int M)
  { int h = 0, a = 127;
    for (; *v != '\0'; v++)
      h = (a*h + *v) % M;
    return h;
  }
```

Why 127 instead of 128?

Binary	01100001	01100010	01100011	01100100
Hex	61	62	63	64
Dec	97	98	99	100
ascii	a	b	c	d

Universal Hashing and collision

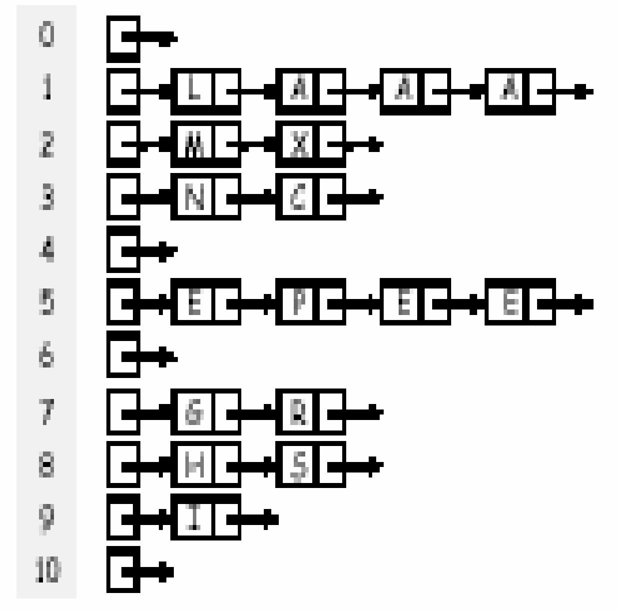
- Universal function
  - Chance of collision for two distinct keys for table size M is precisely 1/M
- How to handle the case when two keys hash to the same value
  - Separate chaining
  - Open addressing –
    - linear probe
    - Double hashing
  - Dynamic hash – increase table size dynamically

```
int hashU(char *v, int M)
  { int h, a = 31415, b = 27183;
    for (h = 0; *v != '\0'; v++,
                a = a*b % (M-1))
        h = (a*h + *v) % M;
    return h;
  }
```

Performs well in practice!

Separate Chaining

- A linked list for each hash address
  - M linked lists
- M much smaller than N
- Property 14.1: Number of comparisons
  - Reduced by factor of M
  - Average length of the lists is N/M
- Search the list
  - Unordered:
    - insert takes constant time
    - Search is proportional to N/M



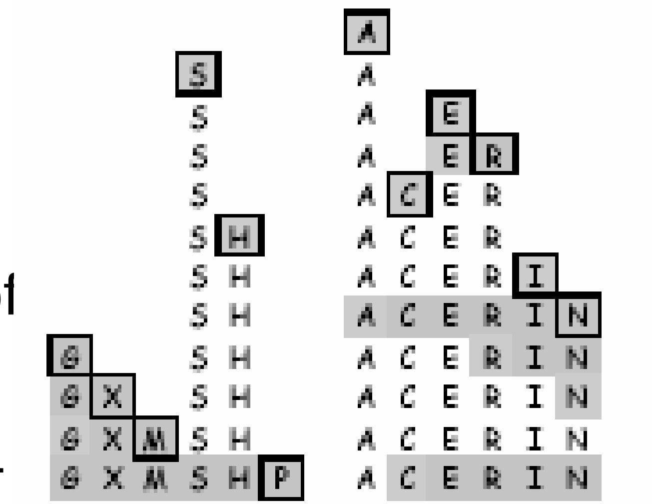
Open Addressing

- Open addressing
  - M is much larger than N
  - Plenty of empty table slots
  - When a new key collides find an empty slot
  - Complex collision patterns
- Linear Probing
  - When collision occurs, check (probe) the next position in the table
    - Wrap around the table to find an empty slot

Linear Probing

A	S	E	R	C	H	I	N	G	X	M	P
7	3	9	9	8	4	11	7	10	12	0	8

- Load factor
  - α - fraction of the table positions that are occupied (less than 1)
    - Search increases with the value of α
    - Search loops infinitely when α = 1
  - Insert: ½(1+ (/(1- α)2)



$$\text{insert: } \frac{1}{2}\left(1+\frac{1}{(1-\alpha)^2}\right)$$

$$\text{search: } \frac{1}{2}\left(1+\frac{1}{(1-\alpha)}\right)$$

Double Hashing

- Avoid clustering using second hash
- Take hash function relatively prime to avoid from probe sequence to be very short
  - Make M prime
  - Choose second has value that returns values less than M
    - A useful second hash: (k mod 97) + 1

insert: $\frac{1}{1-\alpha}$

search: $\frac{1}{\alpha} \ln(1+\alpha)$