Welcome to IS 2610

Introduction

Course Information

- Lecture:
  - James B D Joshi
  - Mondays: 3:00-5.50 PM
    - One (two) 15 (10) minutes break(s)
  - Office Hours: Wed 1:00-3:00PM/Appointment
- Pre-requisite
  - one programming language

Course material

- Textbook
  - *Algorithm in C* (Parts 1-5 Bundle)- Third Edition by Robert Sedgewick, (ISBN: 0-201-31452-1, 0-201-31663-3), Addison-Wesley
- References
  - Introduction to Algorithms, Cormen, Leiserson, and Rivest, MIT Press/McGraw-Hill, Cambridge (Theory)
  - *Fundamentals of Data Structures* by Ellis Horowitz, Sartaj Sahni, Susan Anderson-Freed *Hardcover*/ March 1992 / 0716782502
  - The C Programming language, Kernigham & Ritchie (Programming)
  - Other material will be posted (URLs for tutorials)

Course outline

- *Introduction to Data Structures and Analysis of Algorithms*
  - Analysis of Algorithms
  - Elementary/Abstract data types
  - Recursion and Trees
- *Sorting Algorithms*
  - Selection, Insertion, Bubble, Shellsort
  - Quicksort
  - Mergesort
  - Heapsort
  - Radix sort
- *Searching*
  - Symbol tables
  - Balanced Trees
  - Hashing
  - Radix Search
- *Graph Algorithms*

Grading

- Quiz 10% (in the beginning of the class; on previous lecture)
- Homework/Programming Assignments 40% (typically every week)
- Midterm 25%
- Comprehensive Final 25%

Course Policy

- Your work MUST be your own
  - Zero tolerance for cheating
  - You get an F for the course if you cheat in anything however small – NO DISCUSSION
- Homework
  - There will be penalty for late assignments (15% each day)
  - Ensure clarity in your answers – no credit will be given for vague answers
  - Homework is primarily the GSA’s responsibility
  - Solutions/theory will be posted on the web
- Check webpage for everything!
  - You are responsible for checking the webpage for updates

Overview

- Algorithm
  - A problem-solving method suitable for implementation as a computer program
- Data structures
  - Objects created to organize data used in computation
- Data structure exist as the by-product or end product of algorithms
  - Understanding data structure is essential to understanding algorithms and hence to problem-solving
  - Simple algorithms can give rise to complicated data-structures
  - Complicated algorithms can use simple data structures

Why study Data Structures (and algorithms)

- Using a computer?
  - Solve computational problems?
  - Want it to go faster?
  - Ability to process more data?
- Technology vs. Performance/cost factor
  - Technology can improve things by a constant factor
  - Good algorithm design can do much better and may be cheaper
  - Supercomputer cannot rescue a bad algorithm
- Data structures and algorithms as a field of study
  - Old enough to have basics known
  - New discoveries
  - Burgeoning application areas
  - Philosophical implications?

Simple example

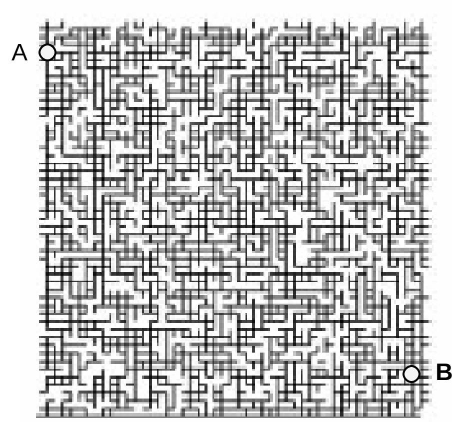
- Algorithm and data structure to do matrix arithmetic
  - Need a structure to store matrix values
    - Use a two dimensional array: `A[M, N]`
  - Algorithm to find the largest element

```
largest = A[0][0];
for (i=0; i < M; i++)
  for (i=0; i < N; i++)
       if (A[i][j]>largest) then
             largest= A[i][j];
```

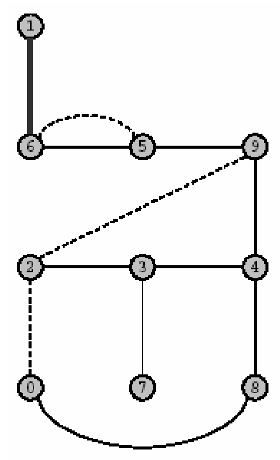
How many times does the `if` statement gets executed?

Another example: Network Connectivity

- Network Connectivity
  - Nodes at grid points
  - Add connections between pairs of nodes
  - Are A and B connected?



Network Connectivity



IN	OUT	Evidence
3 4	3 4	
4 9	4 9	
8 0	8 0	
2 3	2 3	
5 6	5 6	
2 9		(2-3-4-9)
5 9	5 9	
7 3	7 3	
4 8	4 8	
5 6		(5-6)
0 2		(2-3-4-8-0)
6 1	6 1	

Union-Find Abstraction

- What are the critical operations needed to support finding connectivity?
  - N objects – *N can be very large*
    - Grid points
  - FIND: test whether two objects are in same set
    - Is A connected to B?
  - UNION: merge two sets
    - Add a connection
- Define Data Structure to store connectivity information and algorithms for UNION and FIND

Quick-Find algorithm

- Data Structure
  - Use an array of integers – one corresponding to each object
    - Initialize `id[i] = i`
  - If p and q are connected they have the same id
- Algorithmic Operations
  - FIND: to check if p and q are connected, check if they have the same id
  - UNION: To merge components containing p and q, change all entries with `id[p]` to `id[q]`
- Complexity analysis:
  - FIND: takes constant time
  - UNION: takes time proportional to N

```
for (i = 0; i < N; i++)
   id[i] = i;
```

```
if (id[p] == id[q])
   // already connected
```

```
pid = id[p];
for (i = 0; i < N; i++)
   if (id[i] == pid)
      id[i] = id[q];
```

Quick-find

p-q	array entries
3-4	0 1 2 4 4 5 6 7 8 9
4-9	0 1 2 9 9 5 6 7 8 9
8-0	0 1 2 9 9 5 6 7 0 9
2-3	0 1 9 9 9 5 6 7 0 9
5-6	0 1 9 9 9 6 6 7 0 9
5-9	0 1 9 9 9 9 9 7 0 9
7-3	0 1 9 9 9 9 9 9 0 9
4-8	0 1 0 0 0 0 0 0 0 0
6-1	1 1 1 1 1 1 1 1 1 1

Complete algorithm

```
#include <stdio.h>
#define N 10000
main()
{ int i, p, q, t, id[N];
   for (i = 0; i < N; i++) id[i] = i;
   while (scanf("d% %d\n", &p, &q) == 2
        {
                if (id[p] == id[q]) continue;
                for (pid = id[p], i = 0; i < N; i++)
                        if (id[i] == pid) id[i] = id[q];
                printf("s %d\n", p, q);
        }
}
```

- Complexity (M x N)
  - For each of M union operations we iterate for loop at N times

Quick-Union Algorithm

- Data Structure
  - Use an array of integers – one corresponding to each object
    - Initialize `id[i] = i`
  - If p and q are connected they have same root
- Algorithmic Operations
  - FIND: to check if p and q are connected, check if they have the same root

```
for (i = p; i != id[i]; i = id[i]) ;
for (j = q; j != id[j]; j = id[j]) ;
if (i == j) // connected
```

  - UNION: Set the id of the p's root to q's root `id[i] = j;`
- Complexity analysis:
  - FIND: takes time proportional to the depth of p and q in tree
  - UNION: takes constant times

Complete algorithm

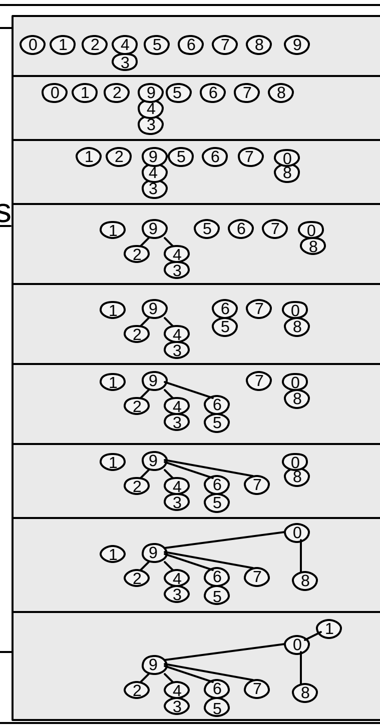
```
#include <stdio.h>
#define N 10000
main()
{ int i, p, q, t, id[N];
   for (i = 0; i < N; i++) id[i] = i;
   while (scanf(“d% %d\n”, &p, &q) == 2
        {       for (i = p; i != id[i]; i = id[i]) ;
                for (j = q; j != id[j]; j = id[j]) ;
                if (i == j) // connected

                id[i] = j;

                printf(“s %d\n”, p, q);
        }
}
```

Quick-Union

p-q	array entries	s
3-4	0 1 2 4 4 5 6 7 8 9	
4-9	0 1 2 4 9 5 6 7 8 9	
8-0	0 1 2 4 9 5 6 7 0 9	
2-3	0 1 9 4 9 5 6 7 0 9	
5-6	0 1 9 4 9 6 6 7 0 9	
5-9	0 1 9 4 9 6 9 7 0 9	
7-3	0 1 9 4 9 6 9 9 0 0	
4-8	0 1 9 4 9 6 9 9 0 0	
6-1	1 1 9 4 9 6 9 9 0 0	



Complexity of Quick-Union

- Less computation for UNION and more computation for FIND
- Quick-Union does not have to go through the entire array for each input pair as does the Union-find
- Depends on the nature of the input
  - Assume input 1-2, 2-3, 3-4,…
  - Tree formed is linear!
- More improvements:
  - Weighted Quick-Union
  - Weighted Quick-Union with Path Compression

Analysis of algorithm

- Empirical analysis
  - Implement the algorithm
  - Input and other factors
    - Actual data
    - Random data (*average-case* behavior)
    - Perverse data (*worst-case* behavior)
  - Run empirical tests
- Mathematical analysis
  - To compare different algorithms
  - To predict performance in a new environment
  - To set values of algorithm parameters

Growth of functions

- Algorithms have a primary parameter N that affects the running time most significantly
  - N typically represents the size of the input– e.g., file size, no. of chars in a string; etc.
- Commonly encounterd running times are proportional to the following functions
  - 1 :Represents a constant
  - *Log N* :Logarithmic
  - *N* :Linear time
  - *N log N* :Linearithmic(?)
  - N^2 :Quadratic
  - N^3 :Cubic
  - 2^N :Exponential

Some common functions

lg N	$N^{0.5}$	*N*	*N lg N*	$N (lg\ N)^2$	N^2	2^N
3	3	10	33	110	100	1042
7	10	100	664	444	10000	$2^{10x10} = 1042^{10}$
10	32	1000	9966	99317	1000000	?
13	100	10000	132877	1765633	100000000	?
17	316	100000	1660964	27588016	10000000000	?
20	1000	1000000	19931569	397267426	1000000000000	?

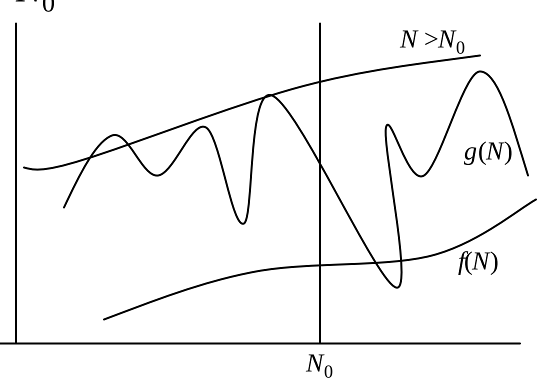
Special functions and mathematical notations

- Floor function : $\lfloor x \rfloor$
  - Largest integer less than or equal to x
  - e.g., $\lfloor 5.16 \rfloor$ = ?
- Ceiling function: $\lceil x \rceil$
  - Smallest integer greater than or equal to x
  - e.g., $\lfloor 5.16 \rfloor$ = ?
- Fibonacci: $F_N = F_{N-1} + F_{N-2}$; *with* $F_0 = F_1 = 1$
  - Find F_2 = ? F_4 = ?
- Harmonic: $H_N = 1 + ½ + 1/3 + \ldots + 1/N$
- Factorial: $N! = N.(N-1)!$
- $\log_e N = \ln N; \log_2 N = \lg N$

Big O-notation – Asymptotic expression

- $g(N) = O(f(N))$ (read $g(N)$ *is said to be* $O(f(N))$) *iff there exist constants* c_0 *and* N_0 *such that* $0 = g(N) = c_0 f(N)$ *for all* $N > N_0$

- Can $N^2 = O(n)$?
- Can $2^N = O(N^M)$?



Big-O Notation

- Uses
  - To bound the error that we make when we ignore small terms in mathematical formulas
    - Allows us to focus on leading terms
    - Example:
      - $N^2 + 3N + 4 = O(N^2)$, *since* $N^2 + 3N + 4 < 2N^2$ *for all* $n > 10$
      - $N^2 + N + N \lg N + \lg N + 1 = O(N^2)$
  - To bound the error that we make when we ignore parts of a program that contribute a small amount to the total being analyzed
  - To allow us to classify algorithms according to the upper bounds on their total running times

$\Omega(f(n))$ and $\Theta(f(n))$

- $g(N) = \Omega(f(N))$ (read $g(N)$ *is said to be* $\Omega(f(N))$) *iff there exist constants* c_0 *and* N_0 *such that* $0 = g(N) = c_0 f(N)$ *for all* $N > N_0$

- $g(N) = \Theta(f(N))$ (read $g(N)$ *is said to be* $\Omega(f(N))$) *iff there exist constants* c_0, c_1 *and* N_0 *such that* $c_1 f(N) = g(N) = c_1 f(N)$ *for all* $N > N_0$

Basic Recurrences

- Principle of recursive decomposition
  - decomposition of problems into one or more smaller ones of the same type
  - Use solutions for the sub-problems to get solution of the problem
- Example 1:
  - Loops through a loop and eliminates one item
  - $C_N = C_{N\text{-}1} + N$, for N =2 with $C_1 = 1$

    $= C_{N\text{-}2} + (N\text{-}1) + N$

    $= C_{N\text{-}3} + (N\text{-}2) + (N\text{-}1) + N$

    …

    $= 1 + 2 + \ldots + (N\text{-}2) + (N\text{-}1) + N = N\,(N+1)/2$
  - *Therefore*, $C_N = O(N^2)$

Basic Recurrences

- Recurrence relations
  - Captures the dependence of the running time of an algorithm for an input of size N on its running time for small inputs
- Example 2:
  - formula for recursive programs for that halves the input in one step
    - $C_N = C_{N/2} + 1$, for $N = 2$ with $C_1 = 1$; let $C_N = lg\ N$, *and* $N = 2^{n.}$

      $= C_{N/2} + 1 + 1$

      $= C_{N/4} + 1 + 1 + 1$

      …

      $= C_{N/N} + n = 1 + n$
  - *Therefore*, $C_N = O(n) = O(lg\ N\,)$

Basic Recurrences

- let $C_N = lg\ N$, *and* $N = 2^n$
  - Show that $C_N = N\ lg\ N$ for
    - $C_N = 2C_{N/2} + N$; for $N = 2$ with $C_1 = 0$;