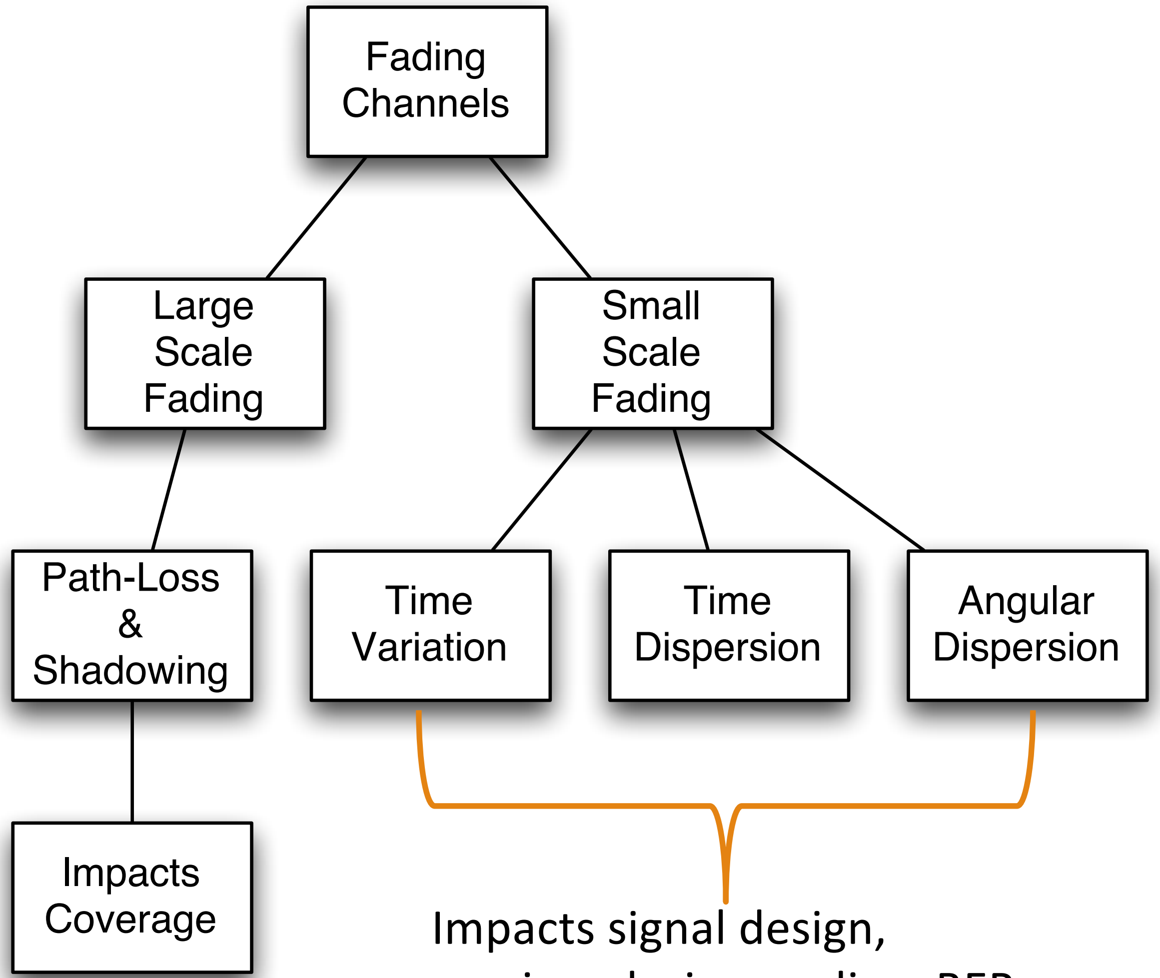
Lecture 7

Traditional Transmission (Narrowband)
Small Scale Fading – Time Variation

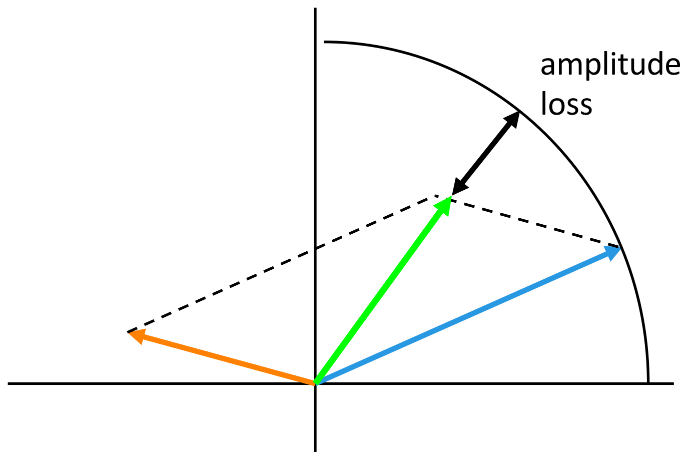
Communication Issues and Radio Propagation

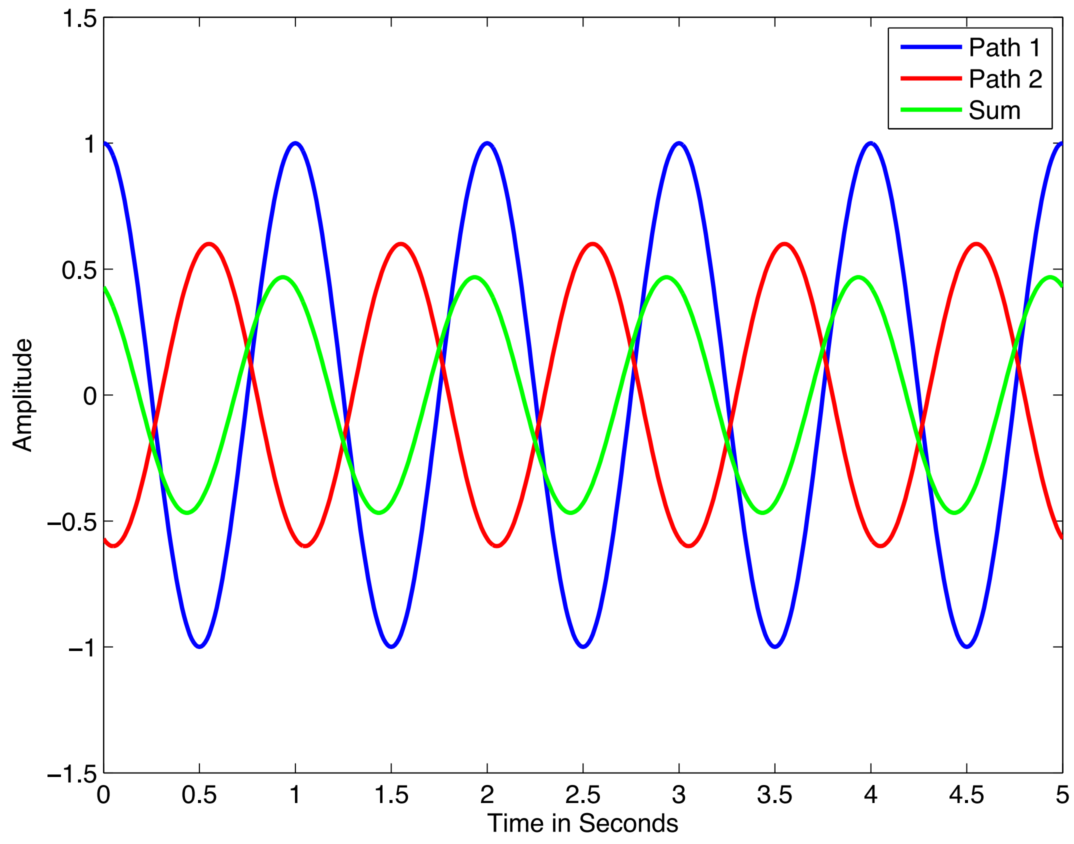


Impacts signal design, receiver design, coding, BER

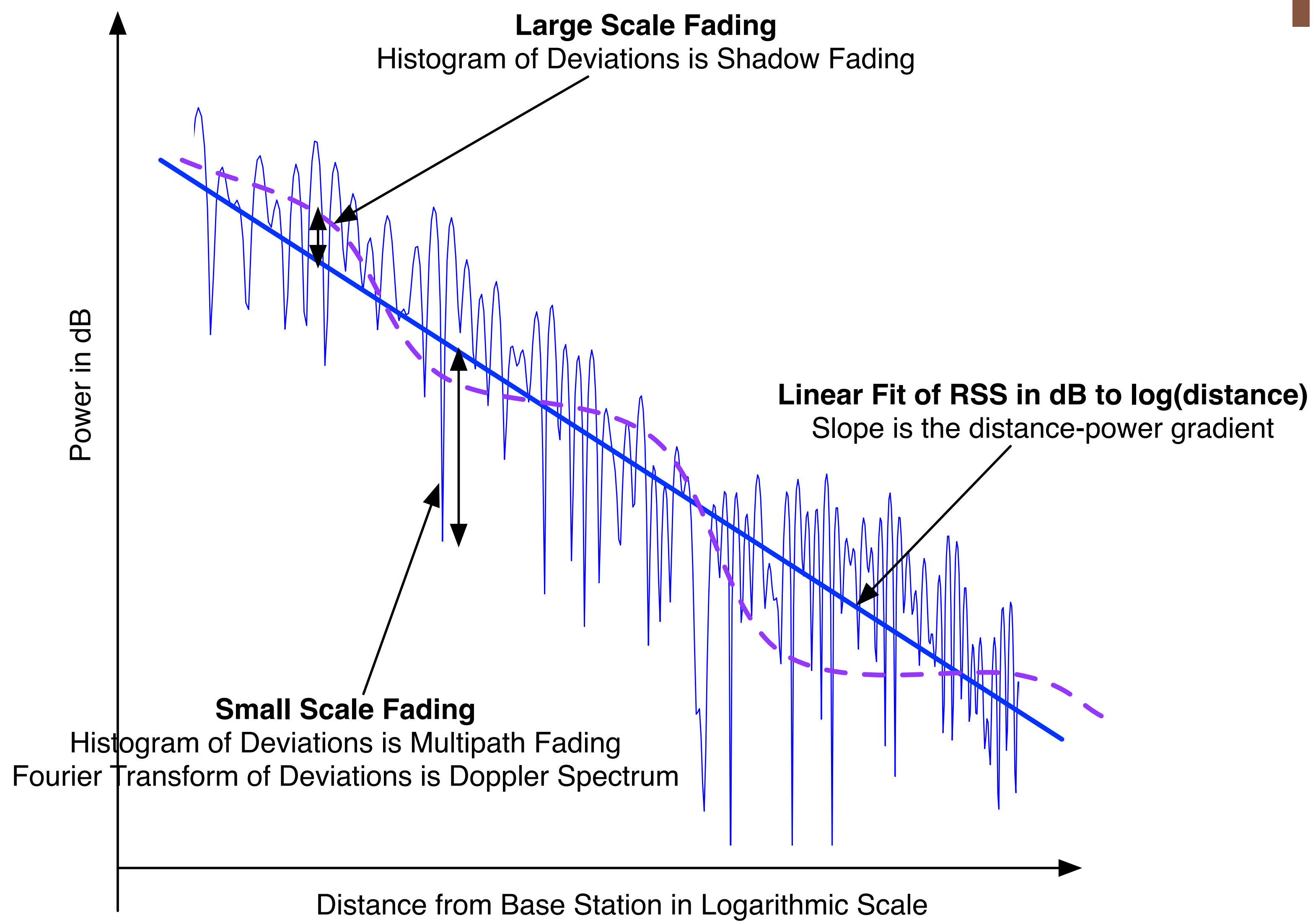
Small scale fading

- Multipath = several delayed replicas of the signal arriving at the receiver
- Fading = constructive and destructive adding of the signals
- Changes with time
- Results in poor signal quality
- Digital communications
  - High bit error rates





Summary

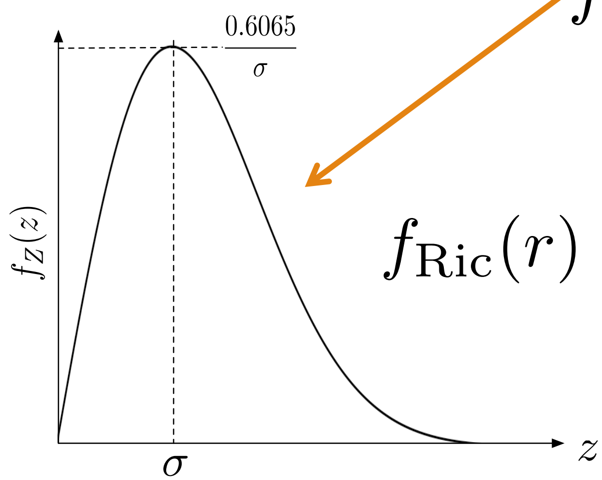

Large Scale Fading
Histogram of Deviations is Shadow Fading
Linear Fit of RSS in dB to log(distance)
Slope is the distance-power gradient
Small Scale Fading
Histogram of Deviations is Multipath Fading
Fourier Transform of Deviations is Doppler Spectrum
Power in dB
Distance from Base Station in Logarithmic Scale

Small scale fading amplitude characteristics

- Amplitudes are Rayleigh distributed
  - Worst case scenario – results in the poorest performance
- In line-of sight situations the amplitudes have a Ricean distribution
  - Strong LOS component has a better performance
  - Weak LOS component tends to a Rayleigh distribution
- Other distributions have been found to fit the amplitude distribution
  - Lognormal
  - Nakagami

Rayleigh, Rician and Lognormal PDFs

$I_0(x)$ is the modified Bessel function of the first kind of order zero



$$f_{\mathrm{Ray}}(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right), r \geq 0$$

$$f_{\mathrm{Ric}}(r) = \frac{r}{\sigma^2} \exp\left(-\frac{r^2 + K^2}{2\sigma^2}\right) I_0\left(\frac{Kr}{\sigma^2}\right), r \geq 0, K \geq 0$$

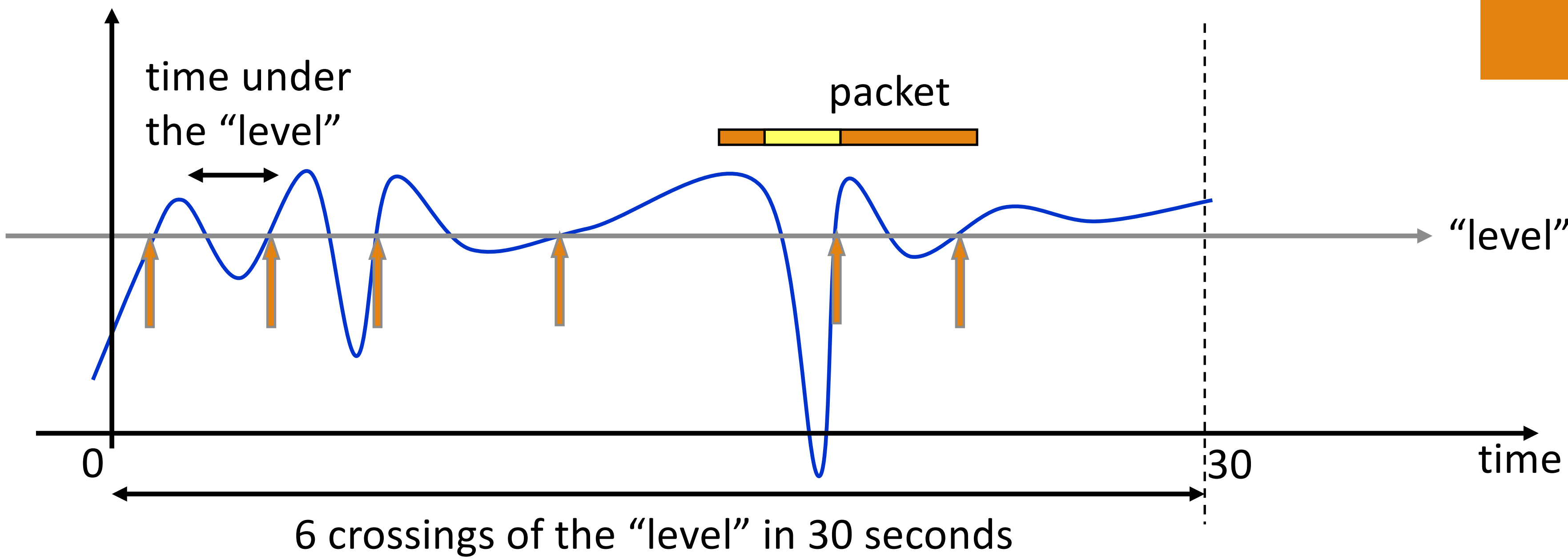
$$f_{\mathrm{LN}}(r) = \frac{1}{\sqrt{2\pi\sigma^2 r}} \exp\left(-\frac{(\ln(r) - \mu)^2}{2\sigma^2}\right)$$

Time variation of the channel

- The radio channel is NOT time invariant
  - Movement of the mobile terminal
  - Movement of objects in the intervening environment
- How quickly does the channel fade (change)?
  - For a time invariant channel, the channel does not change – the signal level is always high or low
  - For time variant channels, it is important to know the rate of change of the channel (or how long the channel is constant)
  - Maximum **Doppler frequency** $f_m = f_c\, v/c$

v is the velocity of the mobile (or speed of changes in the environment)

Fade rate and fade duration



- The signal “level” is the dB above or below the RMS value
- Fade rate determines how quickly the amplitude changes (frequency → Doppler Spectrum)
- Fade duration tells us how long the channel is likely to be “bad”
- Design error correcting codes and interleaving depths to correct errors caused by fading

Fade rate and duration

Level crossing rate:

$$N_R = \sqrt{2\pi} f_m \bar{r} \exp(-\bar{r}^2) \qquad \bar{r} = r / r_{rms}$$

Average fade duration:

$$\tau = \frac{\exp(-\bar{r}^2) - 1}{\bar{r} f_m \sqrt{2\pi}} \qquad \bar{r} = r / r_{rms}$$

f_m = maximum Doppler shift = $f_c\, v/c$

v is the mobile velocity
c is the speed of light
f_c is the carrier frequency

Coherence Time

- How long can you consider the channel to be constant in time?
- Written as T_c
  - Please don't confuse this with the "chip duration" that has the same symbol
- Example
  - v = 100 km/h

$$\frac{f_m}{f_c} = \frac{100 \times 10^3}{3 \times 10^8 \times 3600} = \frac{1}{108 \times 10^5}$$

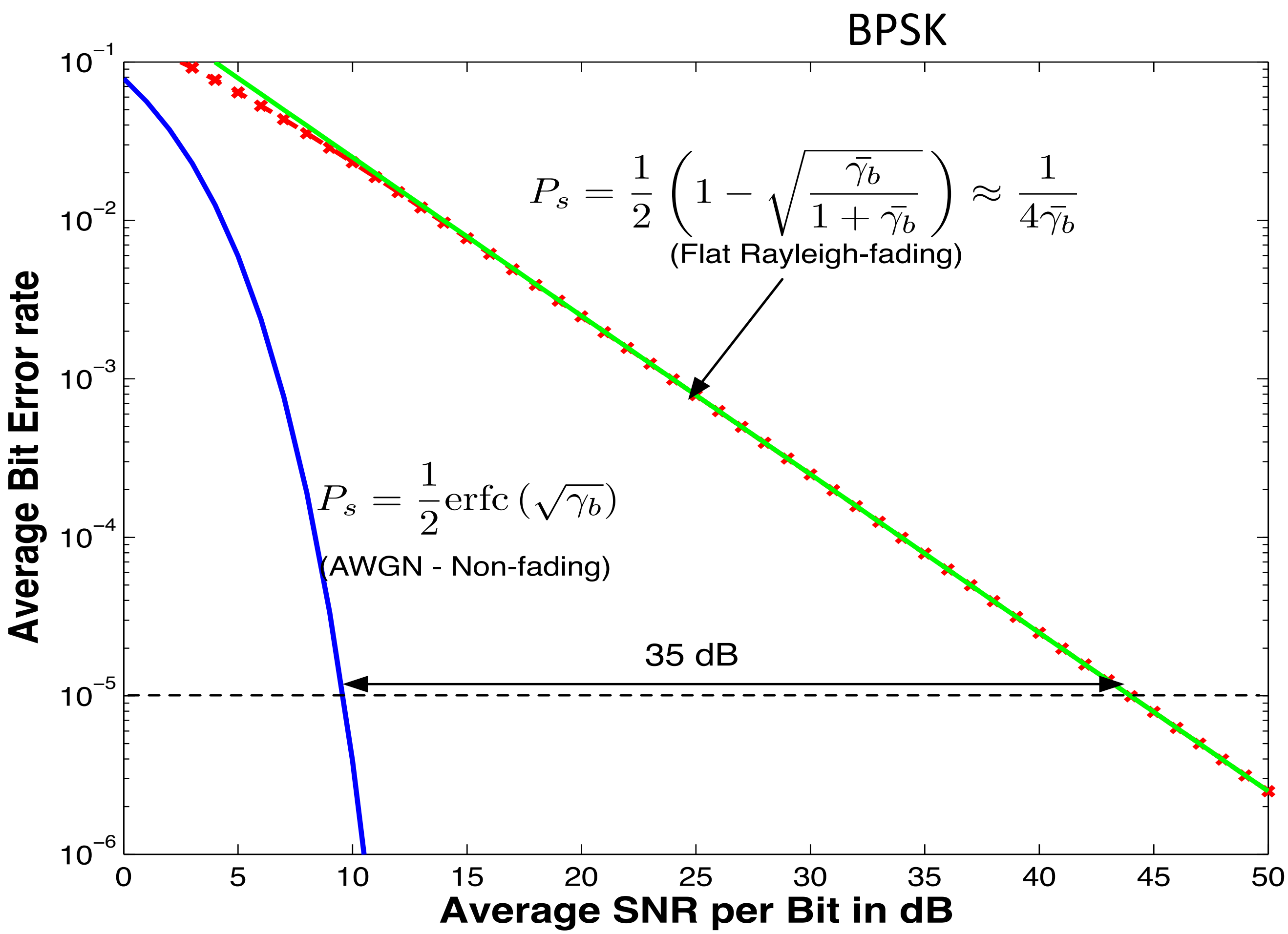
  - At 900 MHz, f_m is about 83.3 Hz
  - The channel changes "could" occur 83.3 times a second
  - T_c= 2.1 ms

$$T_c \approx \frac{9}{16\pi f_m}$$

Performance in Mobile Wireless Channels

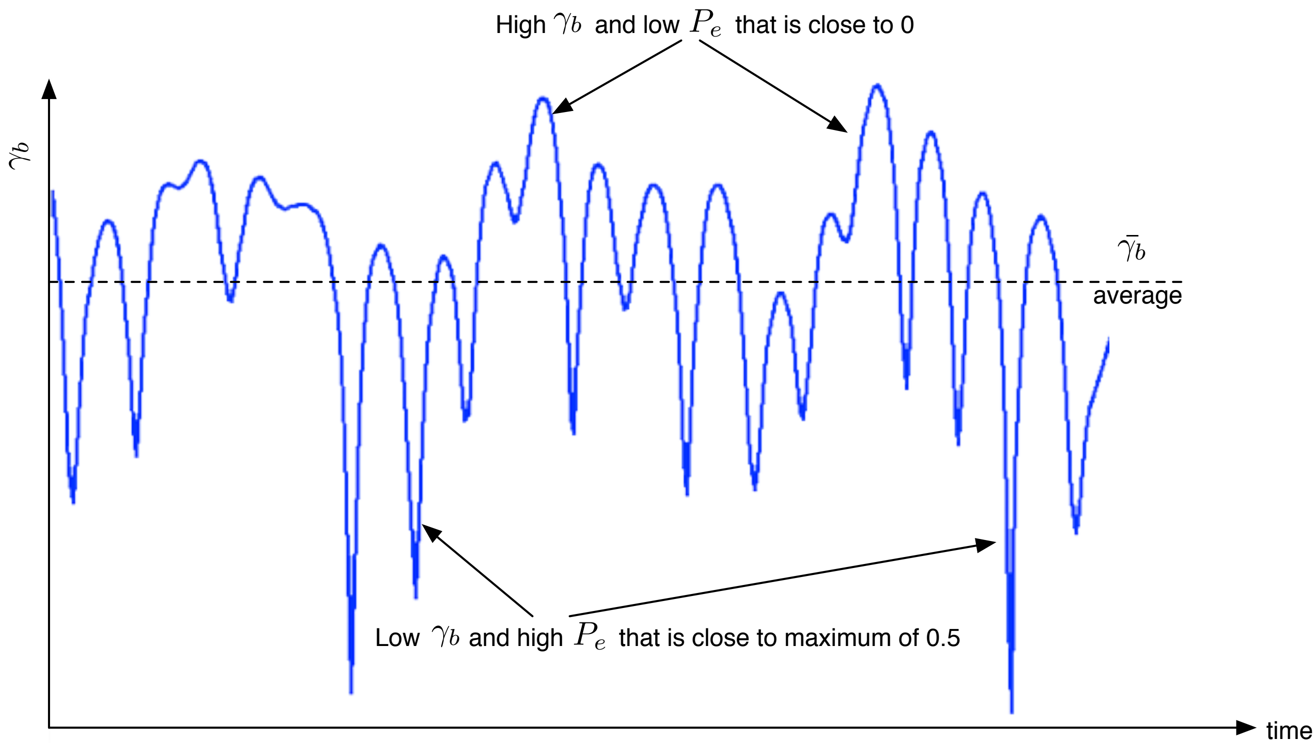
- Wireless channel conditions include
  - Attenuation
  - Multipath
  - Fading
  - Interference
- If the channel is affected by multipath and fading, performance is different from that in AWGN channels
- Ideally we still want
  - Very low bit error rates at low signal to noise ratios under multipath and fading
  - Robust under multipath and fading
    - Does not degrade rapidly if the conditions change
  - Practically, we need an increase in complexity/cost, bandwidth, and/or power to overcome the effects of multipath and fading

Performance in "Flat" Rayleigh Fading Channel



Performance in Flat Rayleigh Fading Channels

- The BER is now a function of the “average” E_b/N_0
- The fall in BER is linear not exponential!
- Large power consumption on average to achieve a good BER
  - 30 dB is three orders of magnitude larger

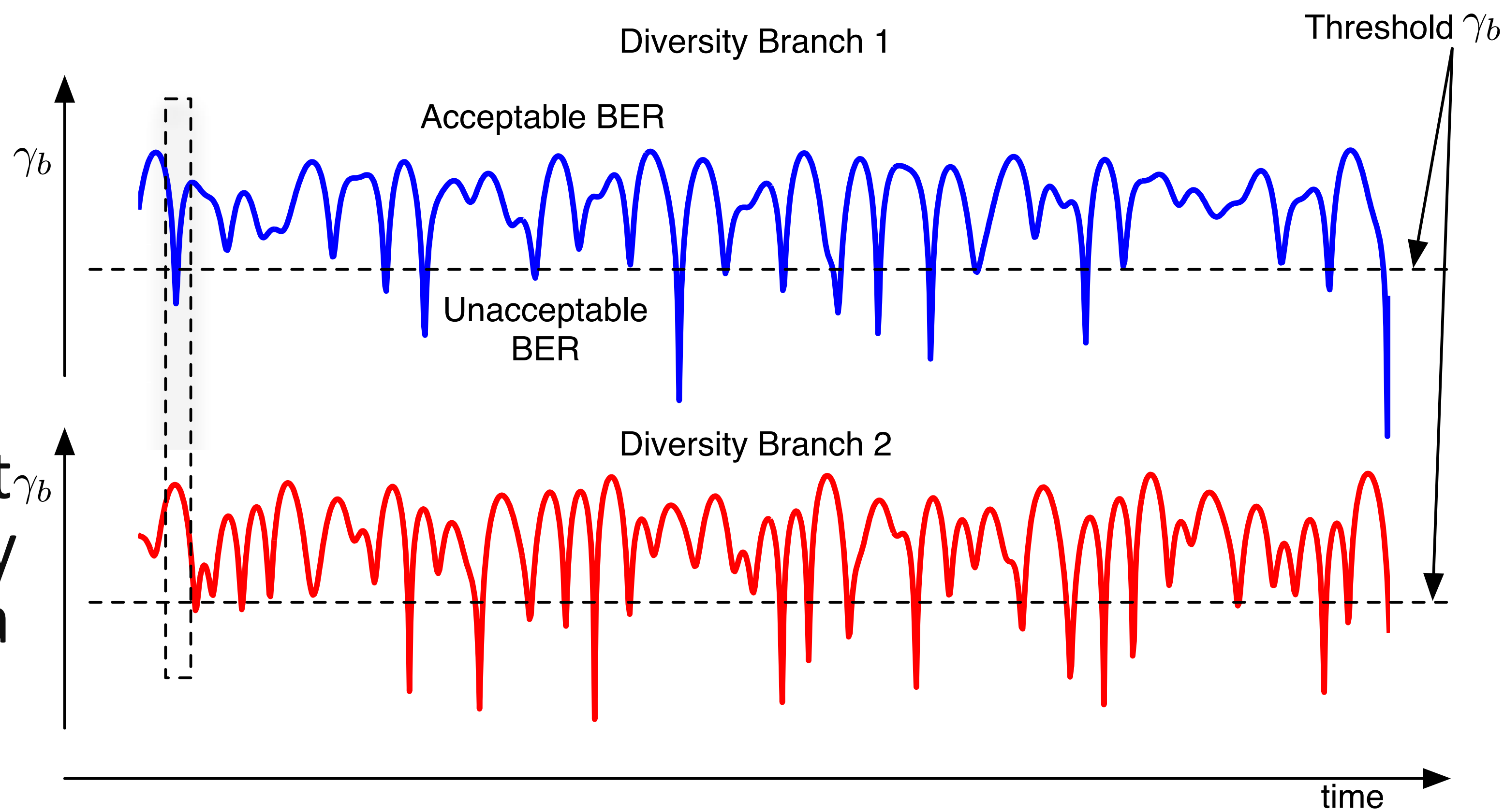


What is diversity?

- Idea: Send the same information over several "uncorrelated" forms
  - Not all repetitions will be lost in a fade
- Types of diversity
  - Time diversity – repeat information in time spaced so as to not simultaneously have fading
    - Error control coding!
  - Frequency diversity – repeat information in frequency channels that are spaced apart
    - Frequency hopping spread spectrum and OFDM
  - Space diversity – use multiple antennas spaced sufficiently apart so that the signals arriving at these antennas are not correlated
    - Usually deployed in all base stations but harder at the mobile
    - Transmit diversity and MIMO
  - Polarization diversity

Example of Diversity

- Note that fades are NOT aligned in time
- Recovering information from at least one diversity branch has a better chance

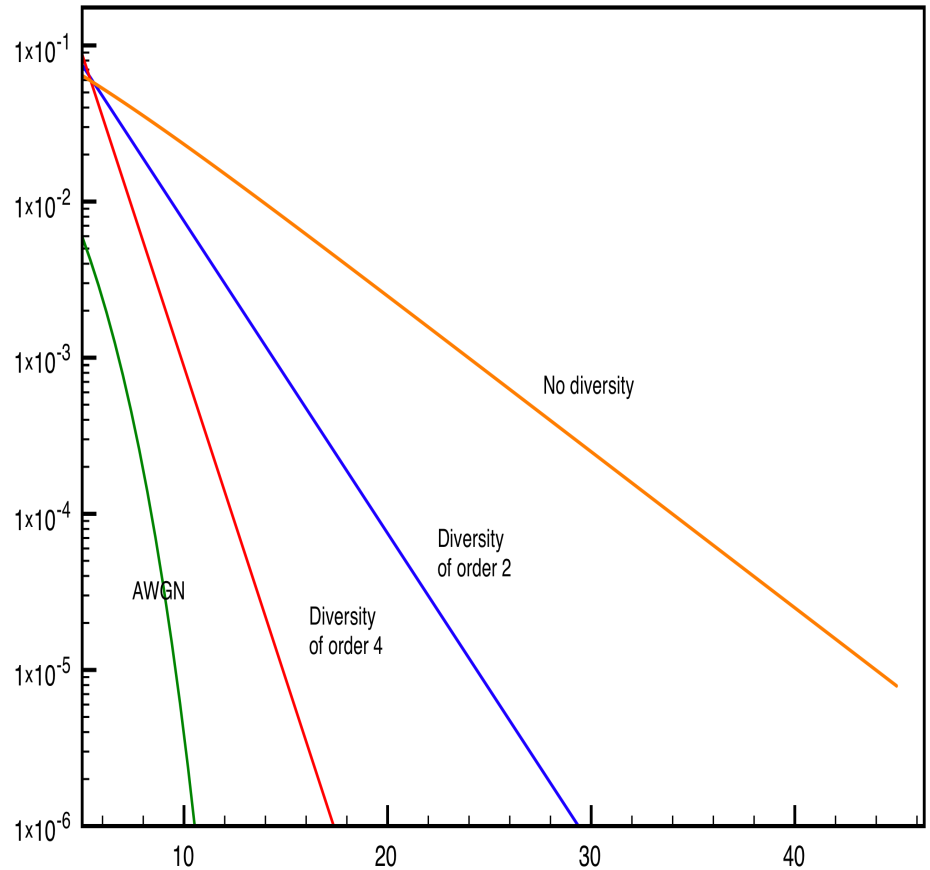


Performance with diversity

- If there is ideal diversity, the performance can improve drastically
- There are different forms of diversity combining
  - Maximal ratio combining
    - Difficult to implement
  - Equal gain combining
    - Easy to implement
  - Selection diversity
    - Easy to implement

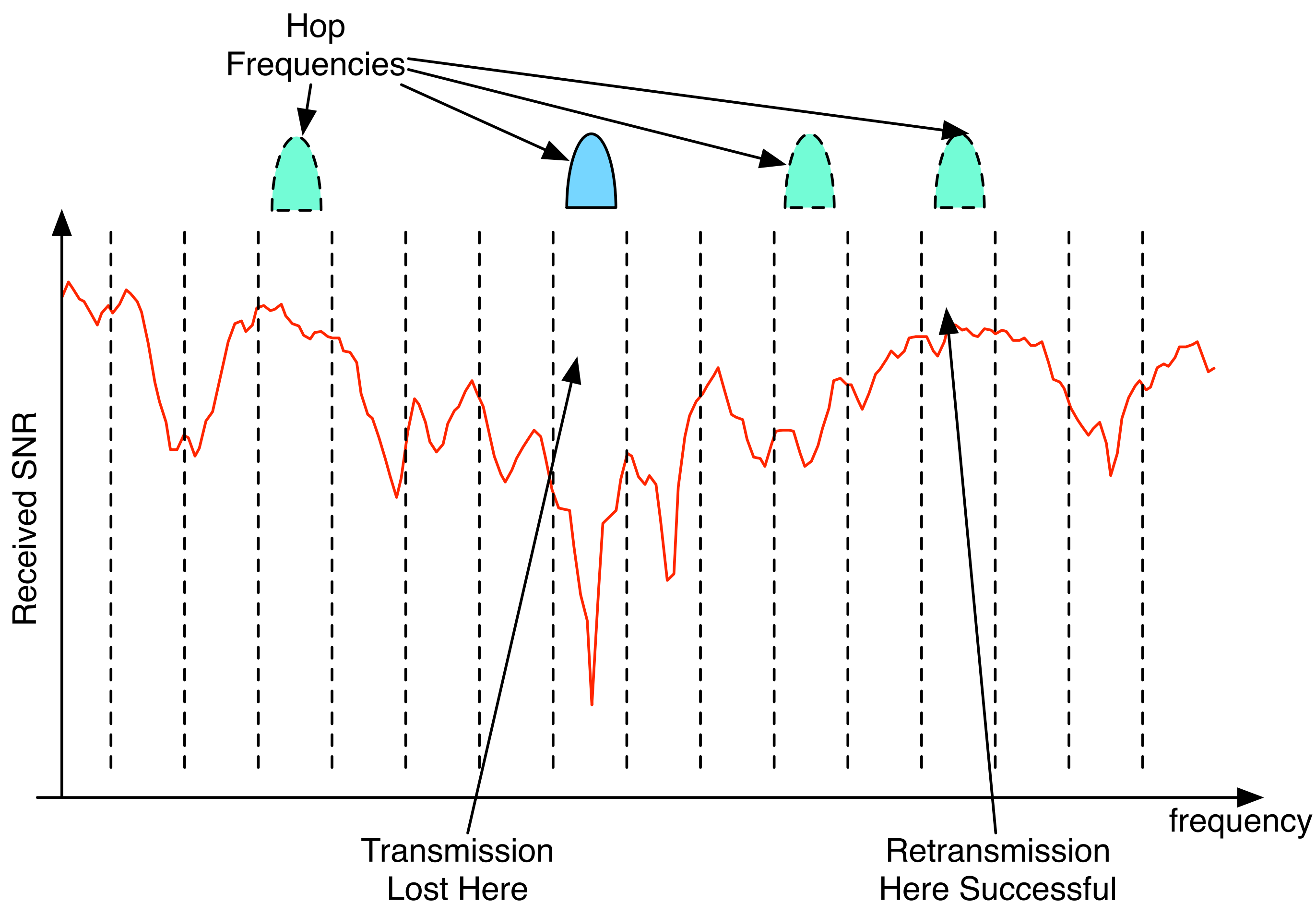
- **Problems**
  - **Bandwidth!**

BPSK with *M* orders of diversity



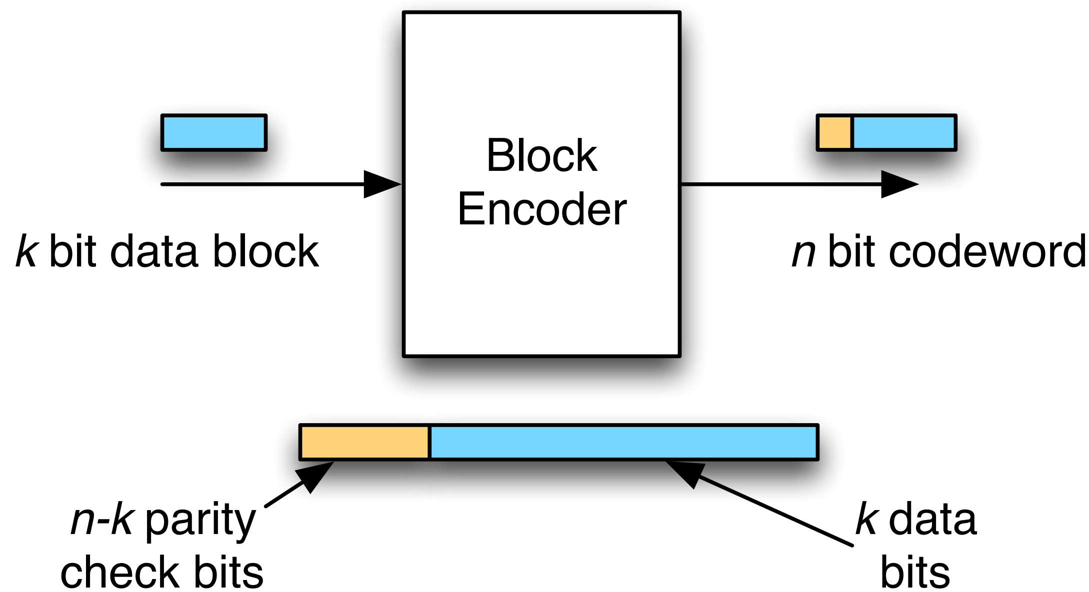
Frequency Hopping and Diversity

- Notice that retransmissions are likely to succeed
- Each transmission occupies a BW < **coherence BW** (later)



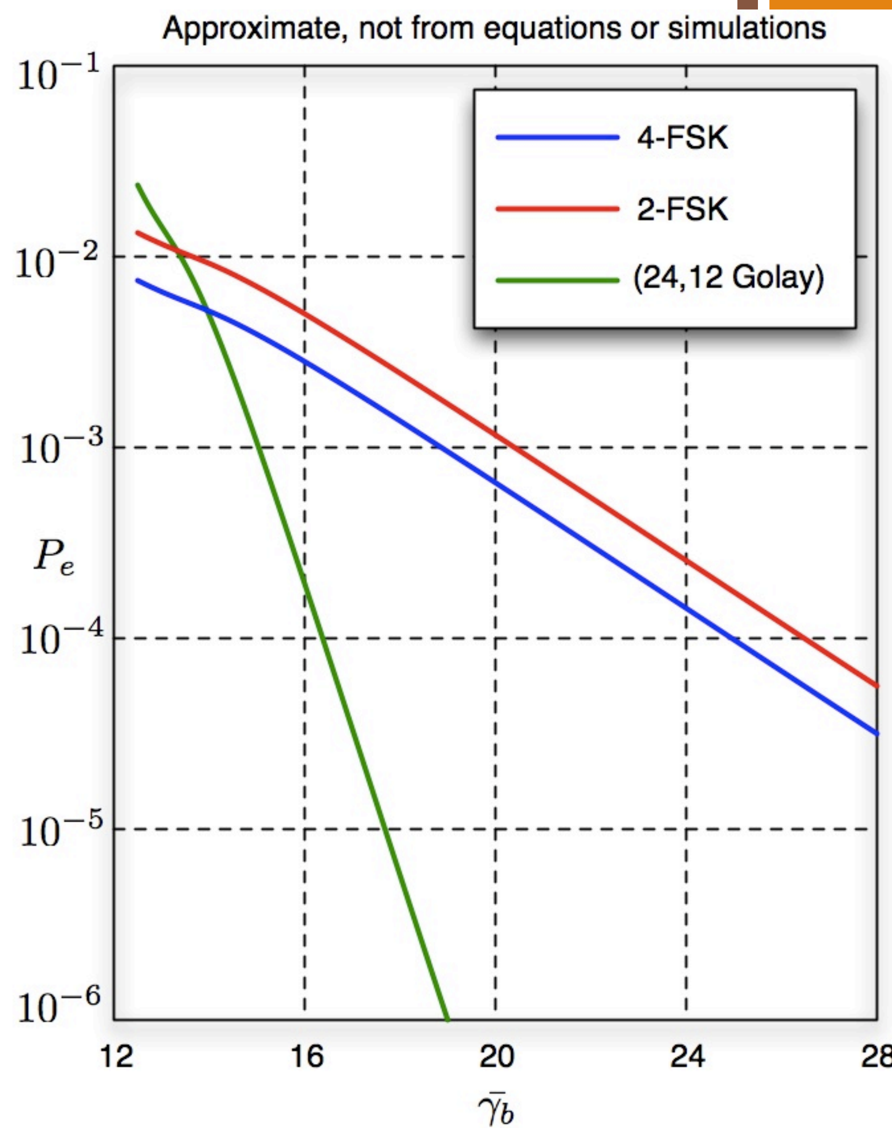
Error control coding

- Coding is a form of diversity
  - Transmit redundant bits using which you can recover from errors
  - The redundant bits have a pattern that enables this recovery
- Types of coding
  - Block codes (n,k)
  - Convolutional codes
  - Trellis coded modulation
  - Turbo codes
- Idea of "code rate" R_c
  - Tradeoffs



Motivation for Error Control Coding

- We cannot derive the performance of error control codes here
- Example of a (24,12) Golay code
  - Rayleigh fading channel
    - BFSK with two orders of diversity
    - BFSK with Golay code



Operation of block codes and interleaving

- Block codes can correct up to t errors in a block of n bits
  - The value of t depends on the code design
    - Hamming codes can correct one error
    - If the *minimum distance* of the code is d_{min}, then the code can
      - Correct $t = \lfloor (d_{min} - 1)/2 \rfloor$ errors
      - Detect $d_{min} - 1$ errors
  - If there are more than t errors, the errors cannot be usually corrected
- In radio channels we see “bursts” of errors that may result in more than t bits in a block of n bits being in error
- In order to correct these burst errors, it is common to “interleave” the bits
  - After coding
  - Before transmitting

What does coding get you?

- Consider a wireless link
  - Probability of a bit error = *q*
  - Probability of correct reception = *p = 1 - q*
  - In a block of *k* bits with no error correction
    - *P(word correctly received) =* p^k
    - *P(word error) = 1 –* p^k
  - With error correction of *t* bits in block of *n* bits

$$P(\text{word correct}) = \sum_{i=0}^{t} \binom{n}{i} (p)^{n-i} q^{i}$$

$$P(\text{word error}) = 1 - P(\text{word correct})$$

What does coding get you?

- Example consider (7,4) Hamming Code when BER = q = .01, p = 0.99
  - In a block of 4 bits with no error correction
  - P(word correctly received) = p^k = .9606
  - P(word error) = 1 – p^k = 0.04
  - With error correction of 1 bit in block of 7 bits

$$P(word\ correct) = \sum_{i=0}^{t} \binom{n}{i} (p)^{n-i} q^{i} = p^{7} + \binom{7}{1} (p)^{6} q^{1} = 0.998$$

$$P(word\ error) = 1 - P(word\ correct) = 0.002$$

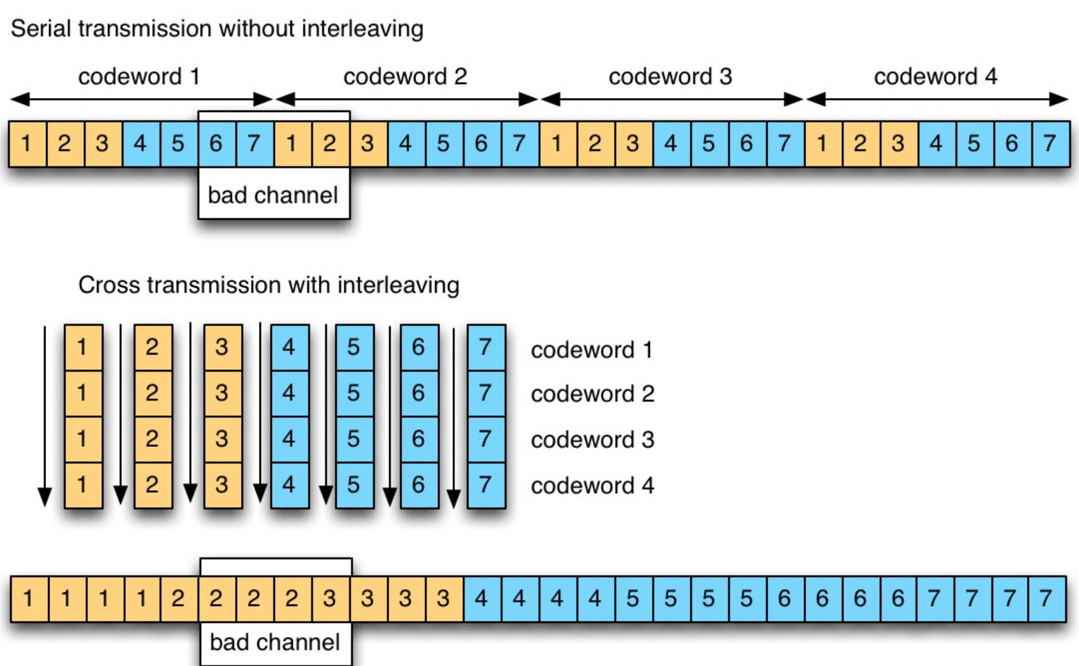
  - Get an order of magnitude improvement in word error rate

Impact of fading and coding

- Problem:
  - An (*n*,*k*) block code consists of codewords that are n-bits long
  - It can correct *t* bit errors within this block of n bits.
- What happens if there is a burst of noise or fade and there are more than *t* bits in error?
  - We have looked at the "average" effect of coding
  - We have ignored the time variation of the channel so far
- Idea:
  - Errors in wireless channels occur in bursts
  - If the errors can be spread over many codewords they can be corrected

Block interleaving

- After codewords are created, the bits in the codewords are interleaved and transmitted
- This ensures that a burst of errors will be dispersed over several codewords and not within the same codeword
- Needs buffering at the receiver to create the original data
- The interleaving depth depends on the nature of the channel, the application under consideration, etc.

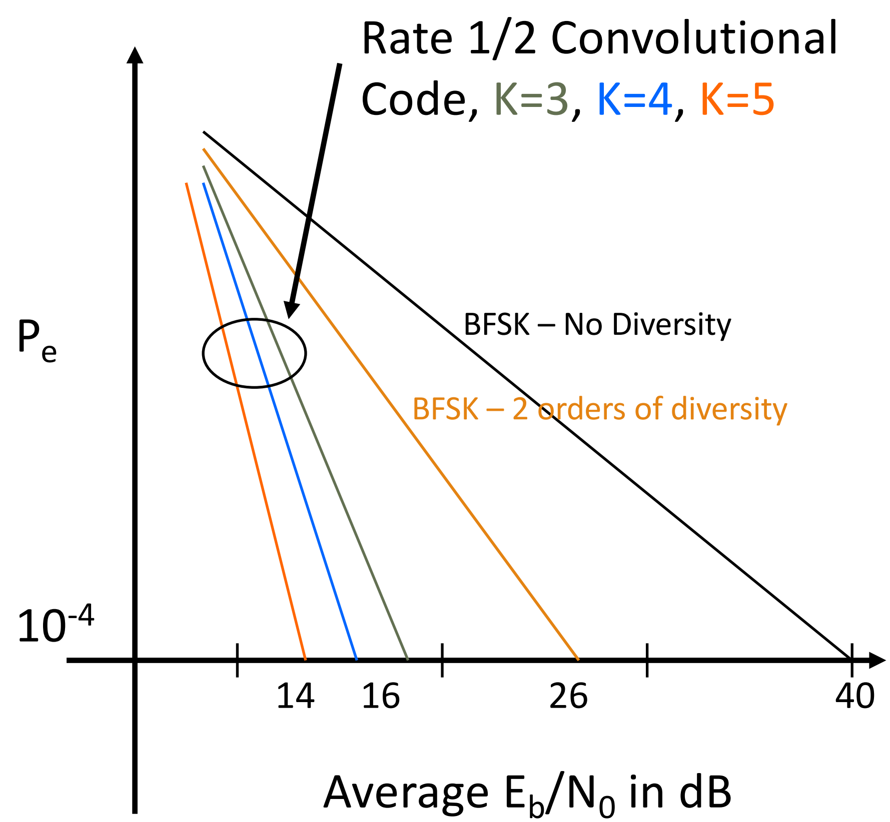


Convolutional Codes

- There is a finite state machine with memory (*K units*) that generates an encoded output from a serial input data
- Decoding is achieved via a “tree” or a “trellis” by choosing the most likely path within the tree or trellis
- Soft decoding is possible
  - A decision on a bit is made based on a variety of signal levels and not a single threshold
- Convolutional codes are more powerful than block codes but they require a larger redundancy
  - Rate 1/3 and ½ codes are used in GSM and CDMA
  - Data rate is reduced by half or two-thirds with these codes

Performance with Convolutional Codes

- Graph is not to scale, but only to give you an idea
- The plot is in a flat Rayleigh fading channel
- You can see that with roughly two orders of diversity, coding is far more efficient



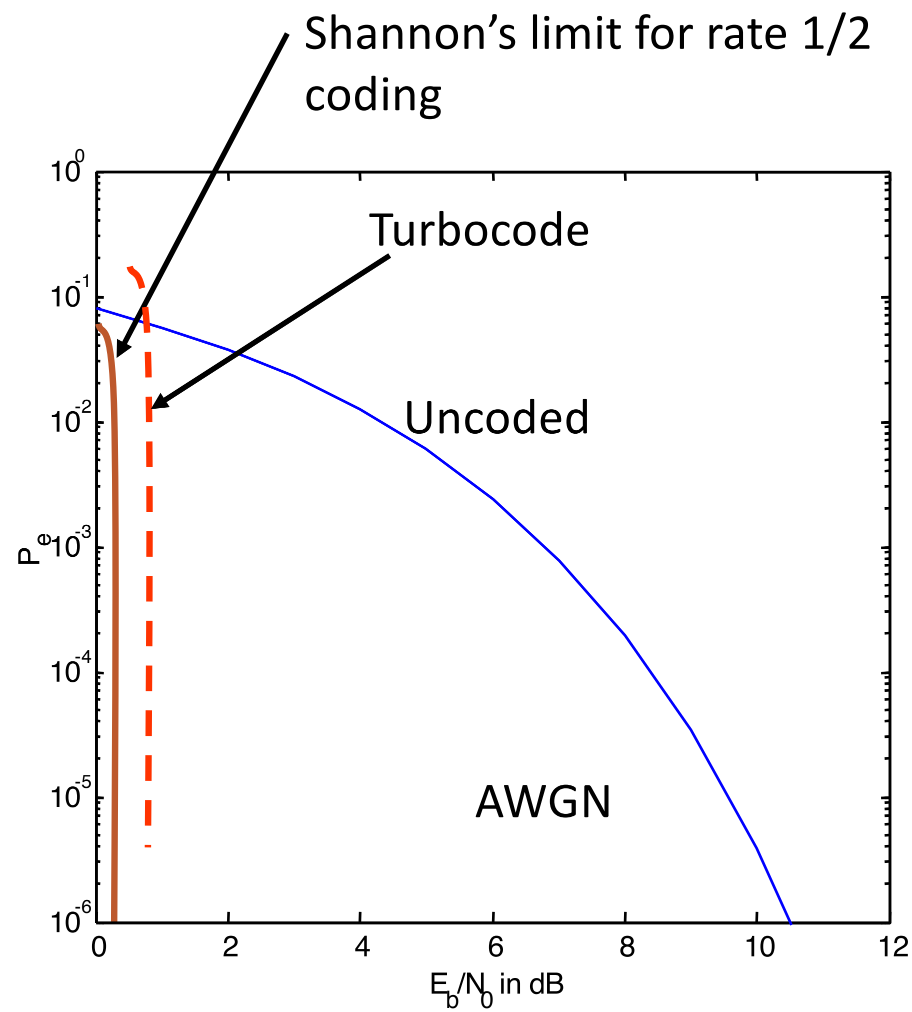
For illustration only; Not to scale

The search for the perfect code

- TurboCode
  - Concatenation of codes with interleaving
    - Followed by an iterative algorithm for decoding
  - Use soft decisions to make the decoding powerful
    - Instead of counting differences in bit positions, distance probabilities are used
    - These are called *probabilistic codes* for this reason unlike typical block and convolutional codes that are called *algebraic codes*
- Used in 3G cellular (UMTS) standard

Turbocode Performance

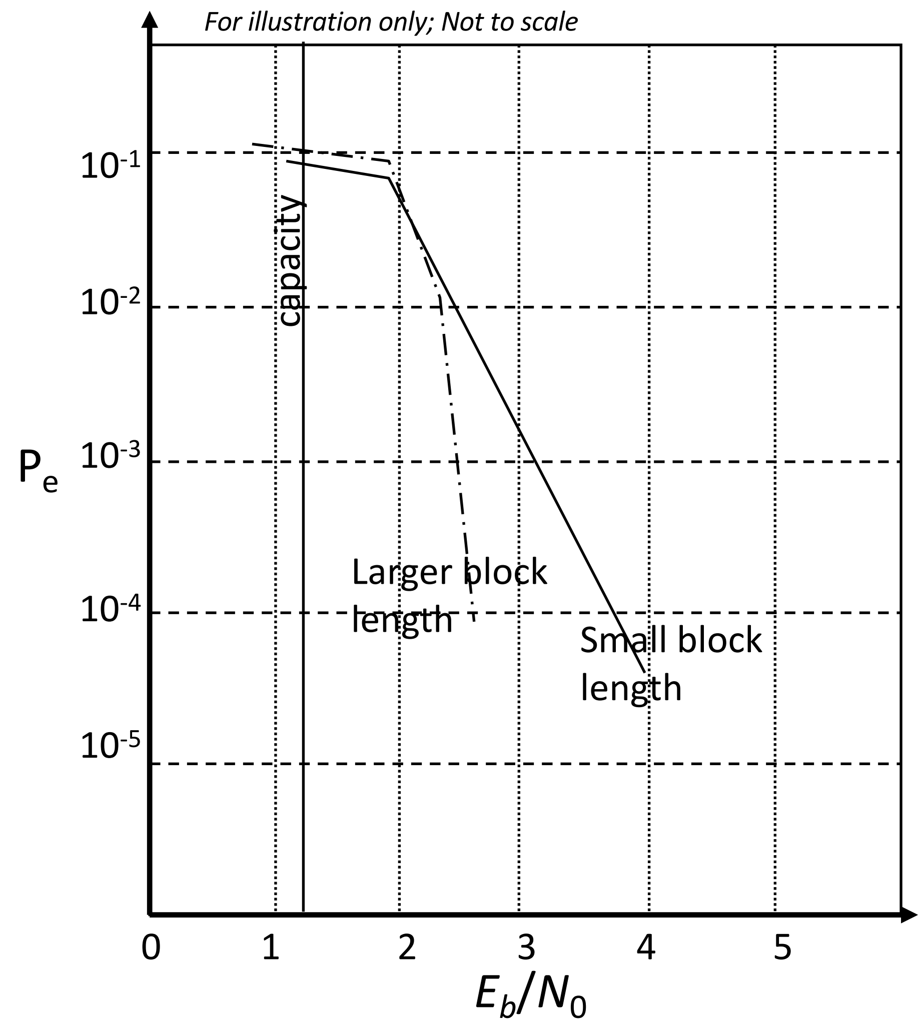
- Once a critical value of E_b/N_0 is reached, the BER with turbocoding drops rapidly
- At $P_e = 10^{-5}$, the turbocode is less than 0.5 dB from Shannon's theoretical limit
  - Needs a large block length
  - Needs a large number of iterations
- It displays an error floor typically at $P_e = 10^{-6}$ or so
  - The dashed curve is halted in the figure



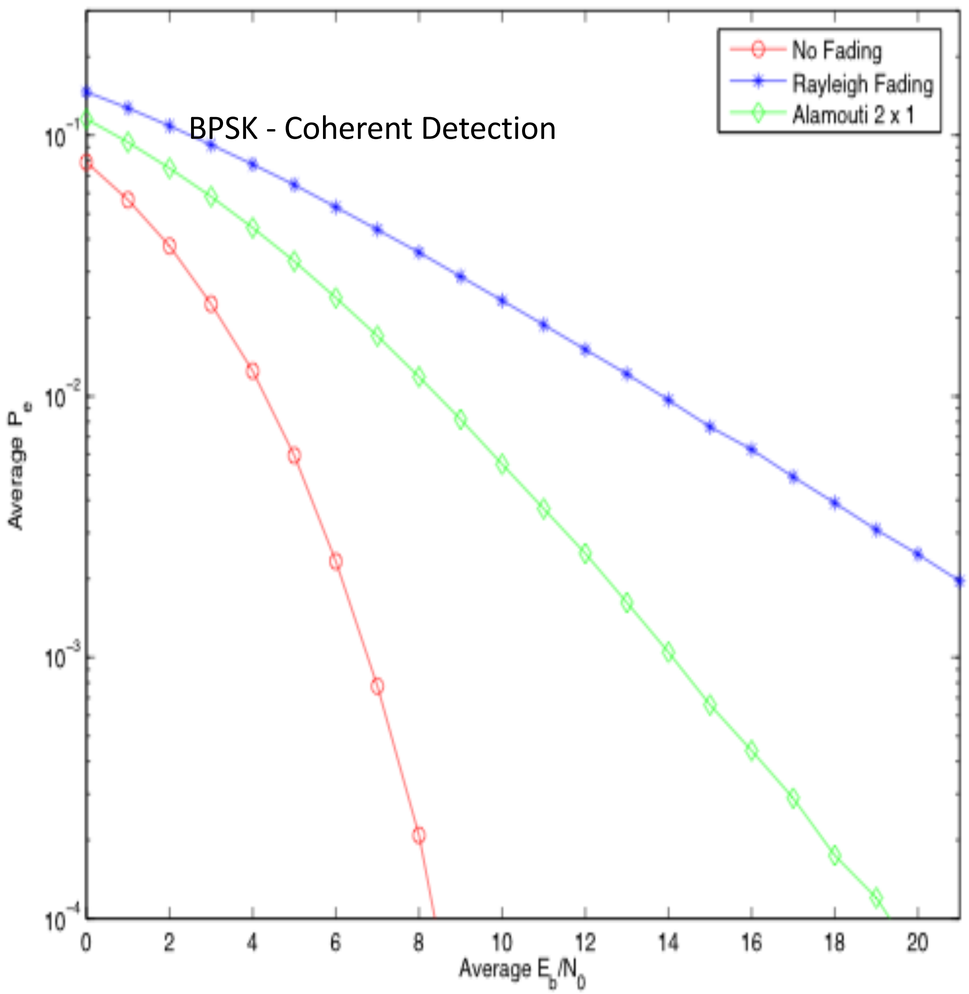
For illustration only; Not to scale

Turbocode Performance in a Flat Rayleigh Fading Channel

- Some results with interleaving and "side information"
- See E.K. Hall and S. G. Wilson, "Design and Analysis of Turbocodes on Rayleigh Fading Channels," *IEEE JSAC*, Feb. 1998



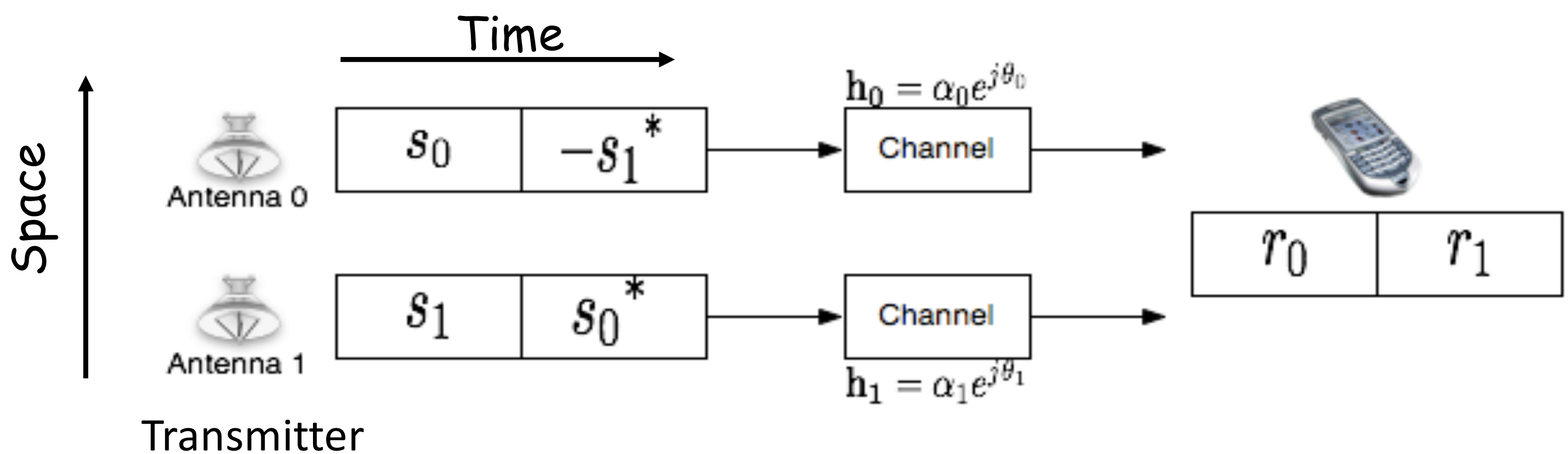
Transmit Diversity: Alamouti Scheme



- Provides close to two orders of diversity
  - It is 3 dB worse than ideal receive diversity because the two transmit antennas split the total power
  - **If there are *M* receive antennas, you can get diversity of order 2*M* in the same way**
  - Works for any complex modulation scheme
- Can think of it as a *"space-time code"*
  - Used in 3G systems

Alamouti's Scheme in a 2×1 system

- Send symbols in space and time as shown below



$$r_0 = h_0 s_0 + h_1 s_1 + n_0$$
$$r_1 = -h_0 s_1^* + h_1 s_0^* + n_1$$

Alamouti's Scheme in a 2×1 system (2)

- Combining scheme

$$\tilde{s_0} = h_0^* r_0 + h_1 r_1^*$$
$$\tilde{s_1} = h_1^* r_0 - h_0 r_1^*$$

- What do we get?

$$\tilde{s_0} = (h_0^* h_0 s_0 + h_0^* h_1 s_1 + h_0^* n_0) + (-h_1 h_0^* s_1 + h_1 h_1^* s_0 + h_1 n_1)$$

$$\Rightarrow \tilde{s_0} = (|\alpha_0^2| + |\alpha_1^2|) s_0 + h_0^* n_0 + h_1 n_1$$

Two orders of diversity

- Similarly

$$\tilde{s_1} = (|\alpha_0^2| + |\alpha_1^2|) s_1 + h_1^* n_0 - h_0 n_1$$

+ MIMO Diversity

- Idea
  - Use ***both*** transmit and receive diversity!
- Consider the Alamouti scheme in the 2×2 MIMO system
  - Send two symbols in two symbol periods
  - Both receive antennas are used to detect the transmitted symbols
- Questions
  - What is the data rate?
  - What is the benefit? (see next)

Alamouti's Scheme in a 2×2 system

- Receive antenna 1 gets:

$$\begin{aligned} r_0 &= h_{11}s_0 + h_{21}s_1 + n_0 \\ r_1 &= -h_{11}s_1^* + h_{21}s_0^* + n_1 \end{aligned}$$

- Receive antenna 2 gets:

$$\begin{aligned} r_2 &= h_{12}s_0 + h_{22}s_1 + n_0 \\ r_3 &= -h_{12}s_1^* + h_{22}s_0^* + n_3 \end{aligned}$$

- Receiver combines signals this way:

$$\begin{aligned} \tilde{s_0} &= h_{11}^*r_0 + h_{21}r_1^* + h_{12}^*r_2 + h_{22}r_3^* \\ \tilde{s_1} &= h_{21}^*r_0 - h_{11}r_1^* + h_{22}^*r_2 - h_{12}r_3^* \end{aligned}$$

What do you end up with?

$$\tilde{s_0} = \left(\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2\right) s_0 + h_{11}^* n_0 + h_{21} n_1^* + h_{12}^* n_2 + h_{22} n_3^*$$
$$\tilde{s_1} = \left(\alpha_0^2 + \alpha_1^2 + \alpha_2^2 + \alpha_3^2\right) s_1 - h_{11} n_1^* + h_{21}^* n_0 - h_{12} n_3^* + h_{22}^* n_2$$

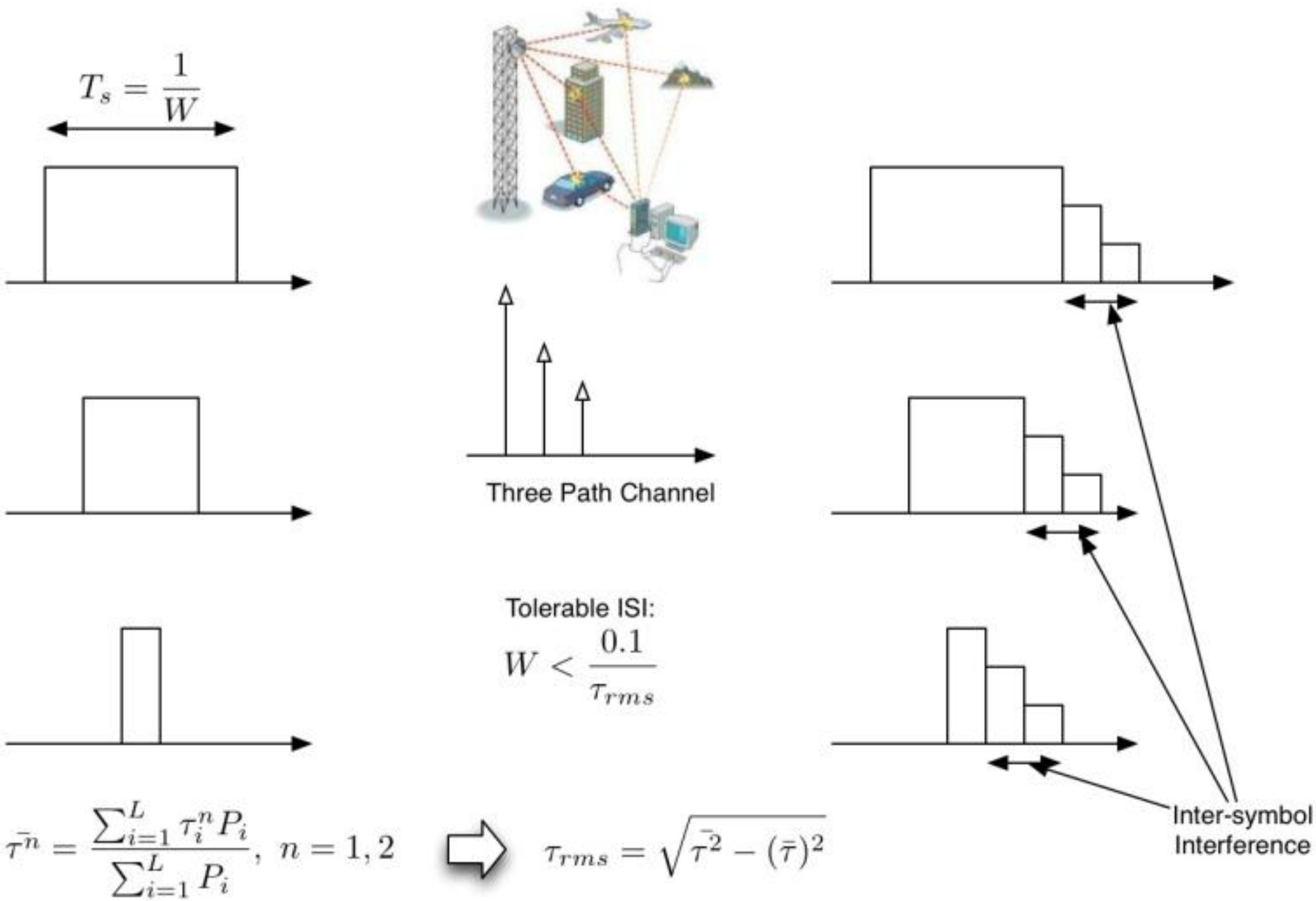
You get 4 orders of diversity!

- You have *both* transmit and receive diversity

Space-Time Block Coding

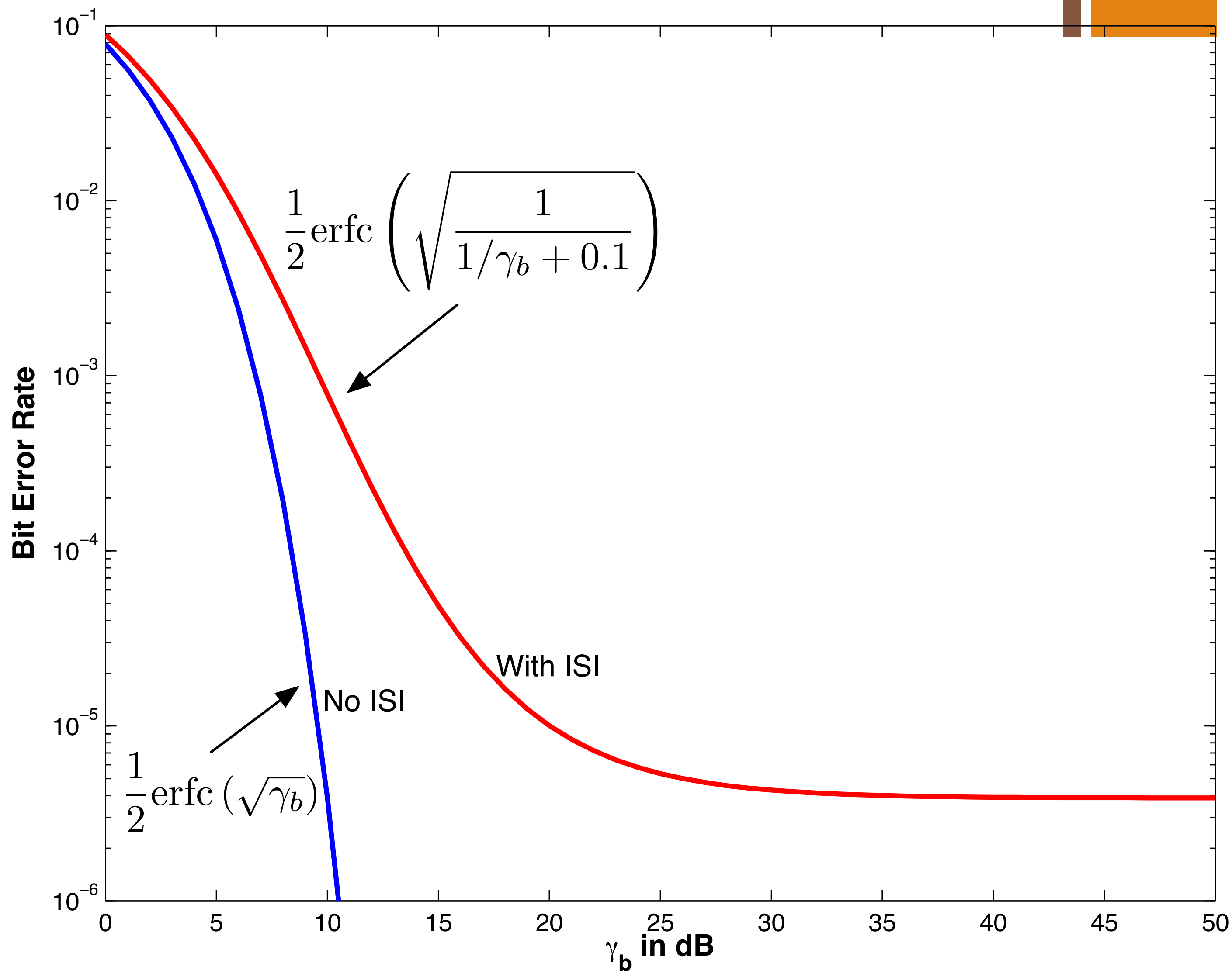
- Generalization of Alamouti codes for any number of transmit antennas
- Parameters:
  - *N* transmit antennas (space)
  - *k* time slots (time)
  - *m* symbols $\{\pm s_0, \pm s_0^*, \pm s_1, \pm s_1^*, \pm s_2, \pm s_2^*, \cdots, \pm s_m, \pm s_m^*\}$
- Rate of the code is $R = m/k$
- Idea: Transmit a "block" of *Nk* symbols with redundancies in space and time
  - Each antenna uses only 1/*N* of the total power

Impact of Time Dispersion



Performance in Frequency Selective Channels

- Figure shows impact of ISI in non-fading channels
- If you include fading, things get worse
- Increasing power has no effect!!



Multipath models for time dispersion

- The time dispersion introduced by the radio channel causes inter-symbol interference and degrades the performance
- The RMS "delay spread" poses a limitation on the maximum data rate that can be supported over a channel
  - Frequency "selective" fading (RMS delay spread > symbol duration)
  - Flat fading (RMS delay spread is << symbol duration)
- Multipath models are required to characterize "wideband" systems
  - TDMA with high data rates
  - CDMA with high chip rates
  - WLANs (many Mbps)

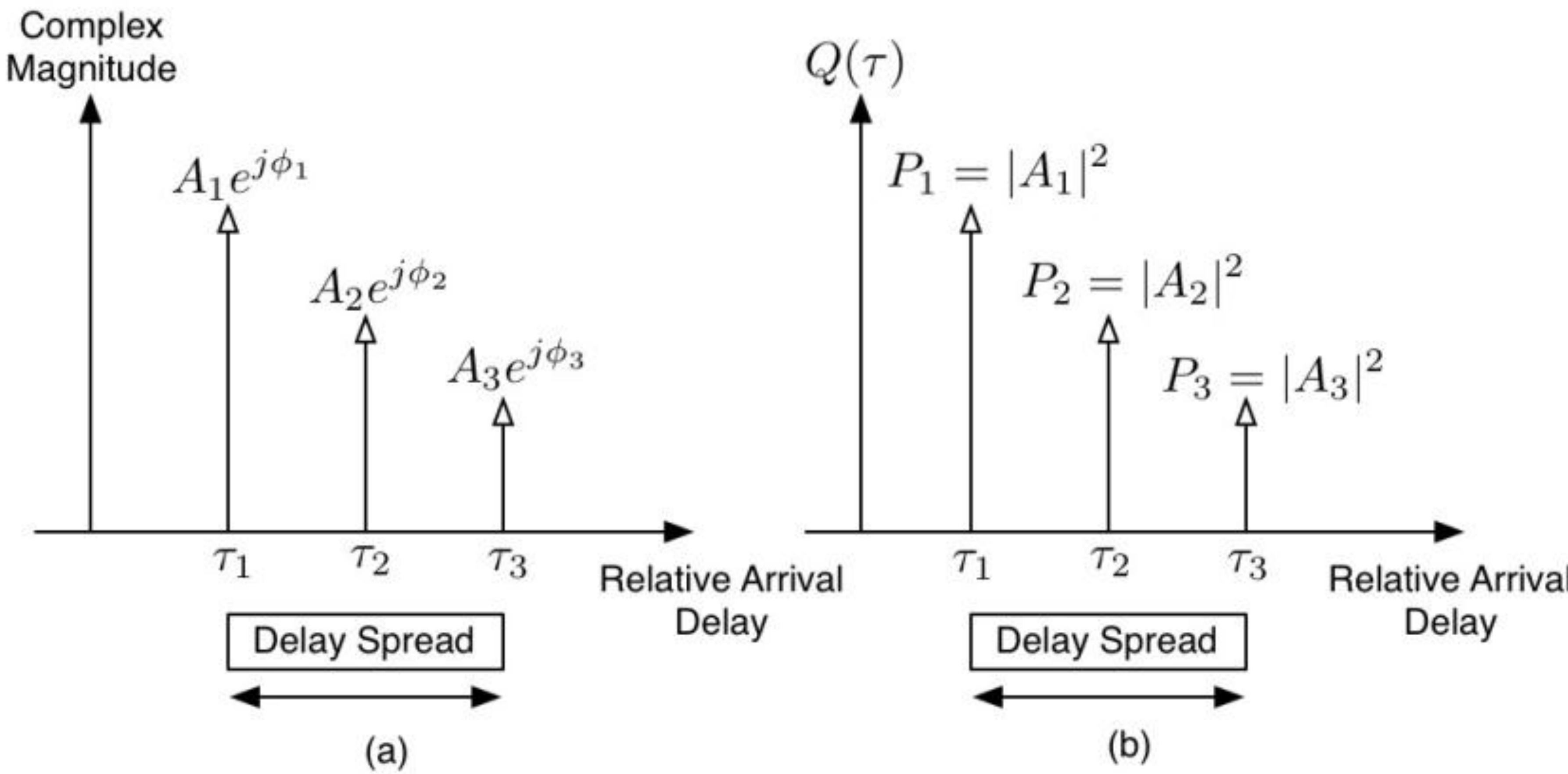
Time dispersion in a radio channel

- Time domain view
- There are multipath components that can cause inter-symbol interference if the symbol duration is smaller than the **multipath delay spread**
- Linear time invariant impulse response

$$Q(\tau) = \sum_{i=1}^{L} P_i \delta(t - \tau_i)$$

- Frequency domain view
- There are multipath components that can cause notches in the frequency response
  - The channel has a **"coherence bandwidth"** where the characteristics are constant
- The coherence bandwidth limits the maximum data rate that can be supported over the channel

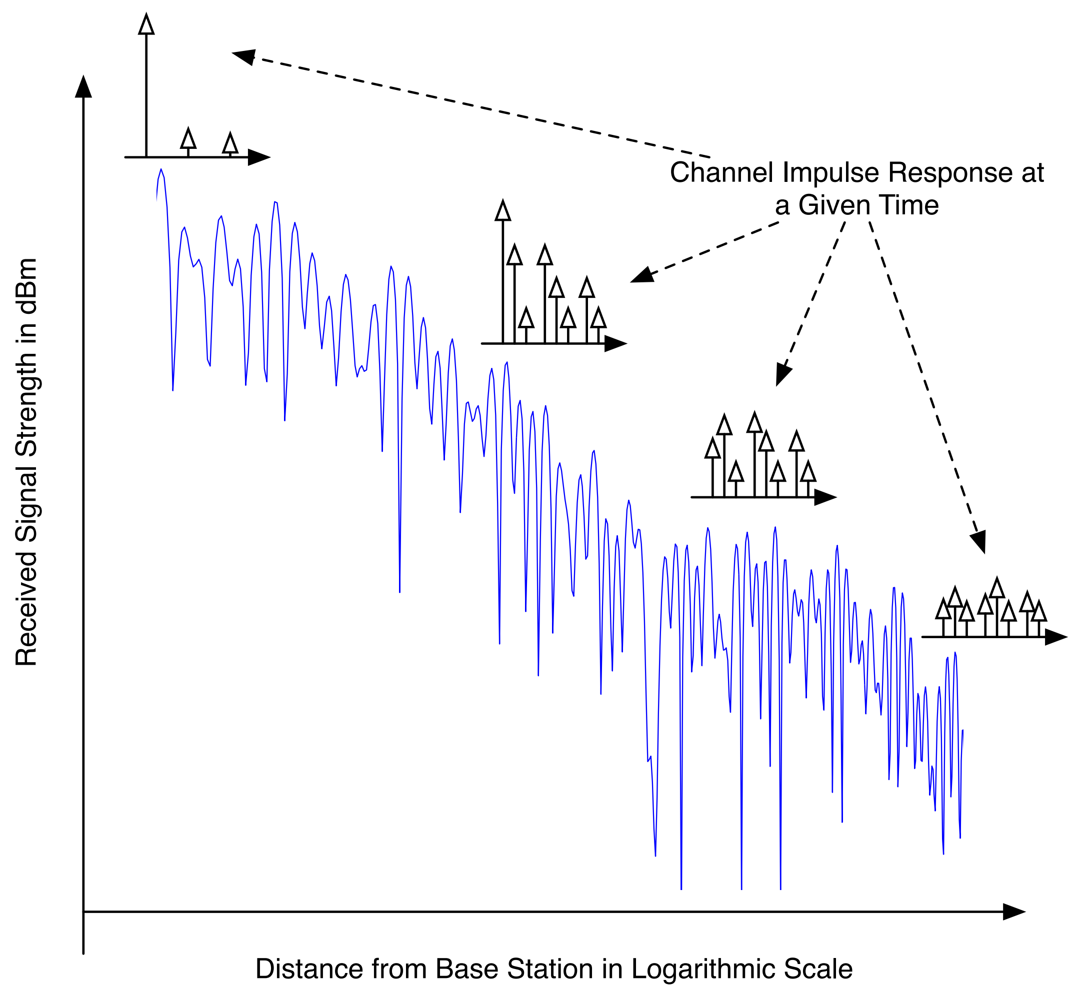
Idea of Delay Spread



Coherence bandwidth of the channel is approximately $1/10\tau_{rms}$

Delay Spread over Distance

But, usually, we assume a constant RMS Delay Spread for a channel

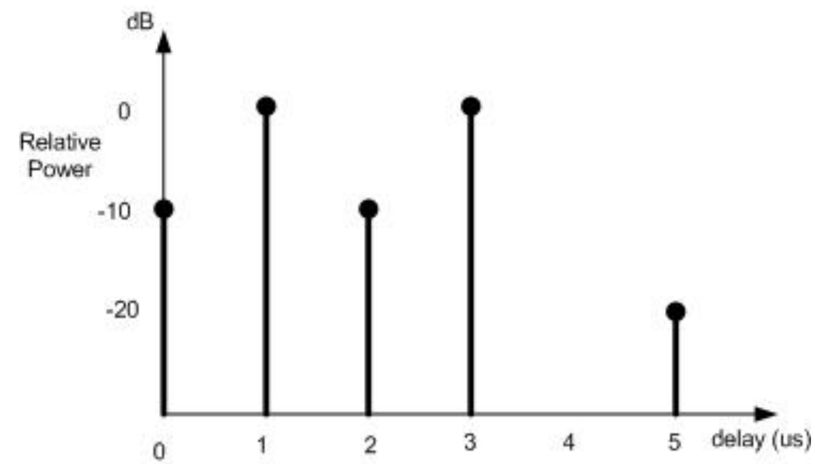


The RMS Delay Spread

- The RMS delay spread is a function of the P_i and τ_i
- The larger the RMS delay spread, the smaller is the data rate that can be supported over the channel
- RMS delay spread varies between a few microseconds in urban areas to a few nanoseconds in indoor areas
  - Higher data rates are possible indoor and not outdoor!!
- The coherence bandwidth determines whether a signal is narrowband or wideband

Example of RMS delay spread

Consider the power delay profile given here



Pl	Delay(us)
0.1	0
1	1
0.1	2
1	3
0.01	5

$$\tau_M = \frac{0.1\times 0 + 1\times 1 + 0.1\times 2 + 1\times 3 + 0.01\times 5}{0.1 + 1 + 0.1 + 1 + 0.01}$$

$$= 1.47\ \mu s$$

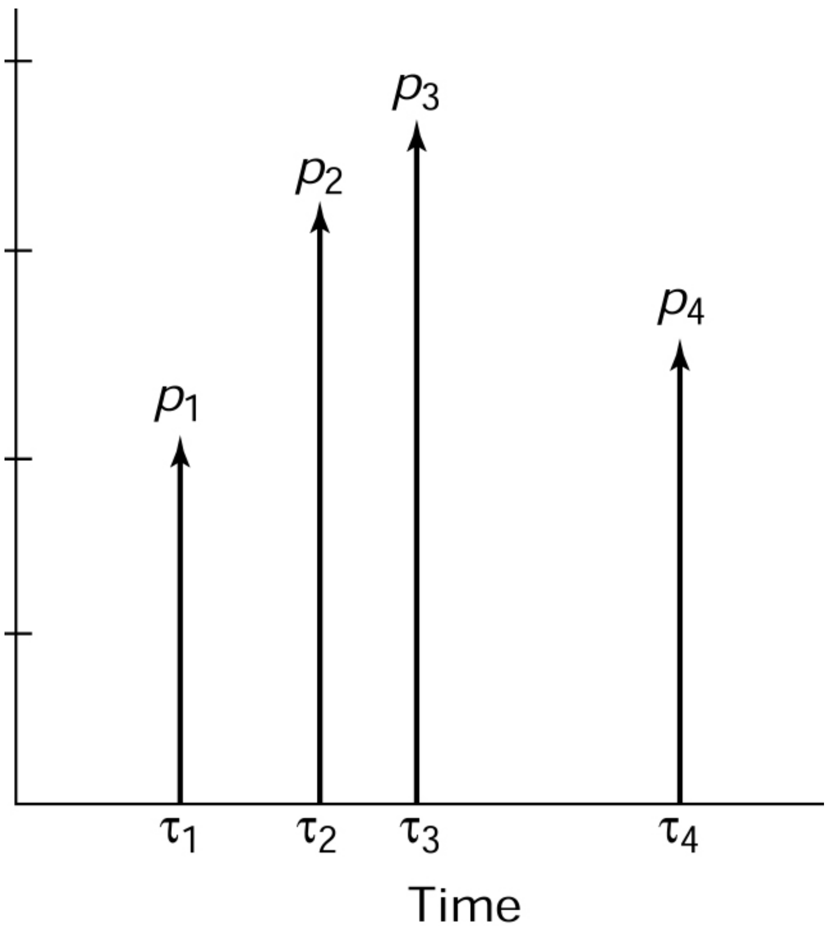
$$\left\langle \tau^2 \right\rangle = \frac{0.1\times 0^2 + 1\times 1^2 + 0.1\times 2^2 + 1\times 3^2 + 0.01\times 5^2}{0.1 + 1 + 0.1 + 1 + 0.01}$$

$$= 4.82\ \mu s^2$$

$$\tau_{RMS} = \sqrt{4.82 - 1.47^2} = 1.39\ \mu s$$

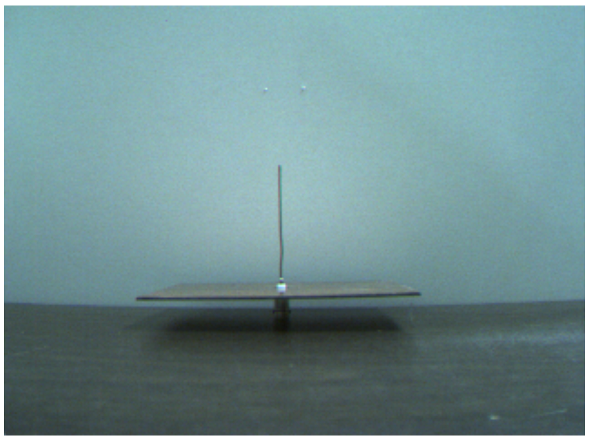
$$\mathrm{B_c} = \frac{1}{10\times 1.39} = 72\ kHz$$

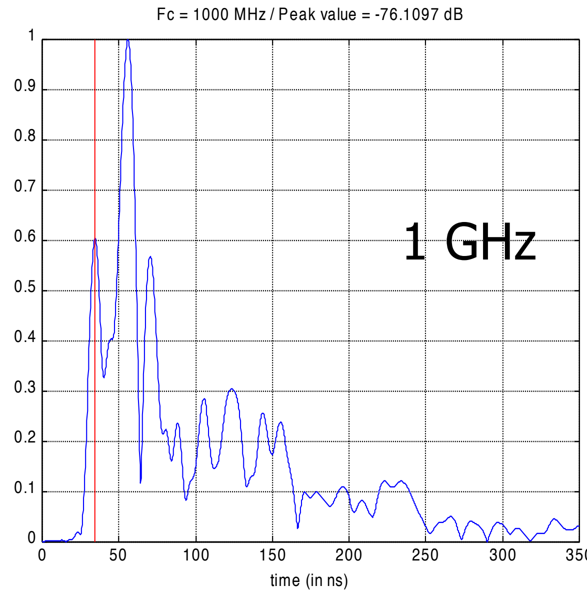
RMS delay spread

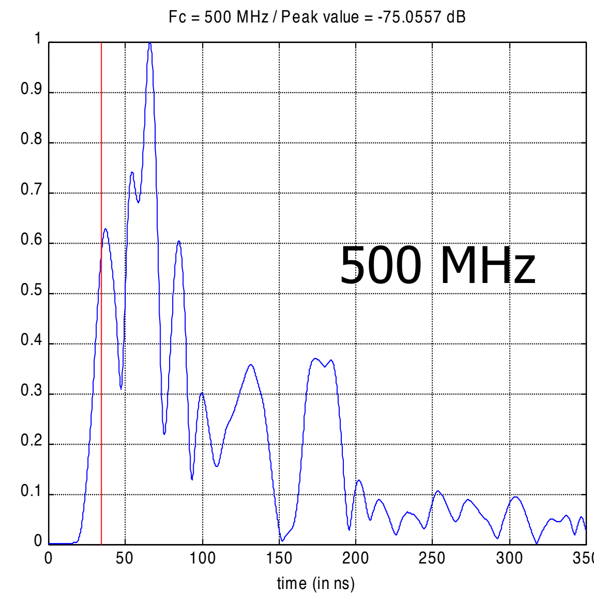


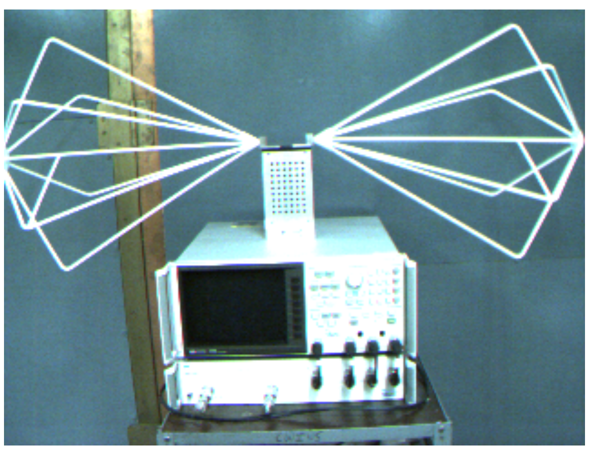
- Measured RMS delay spread values
  - Indoor areas: 30-300 ns
  - Open areas: ≈ 0.2 μs
  - Suburban areas: ≈1 μ s
  - Urban areas: 1-5 μs
  - Hilly urban areas: 3-10 μs

+ Sample measurements – Office Areas

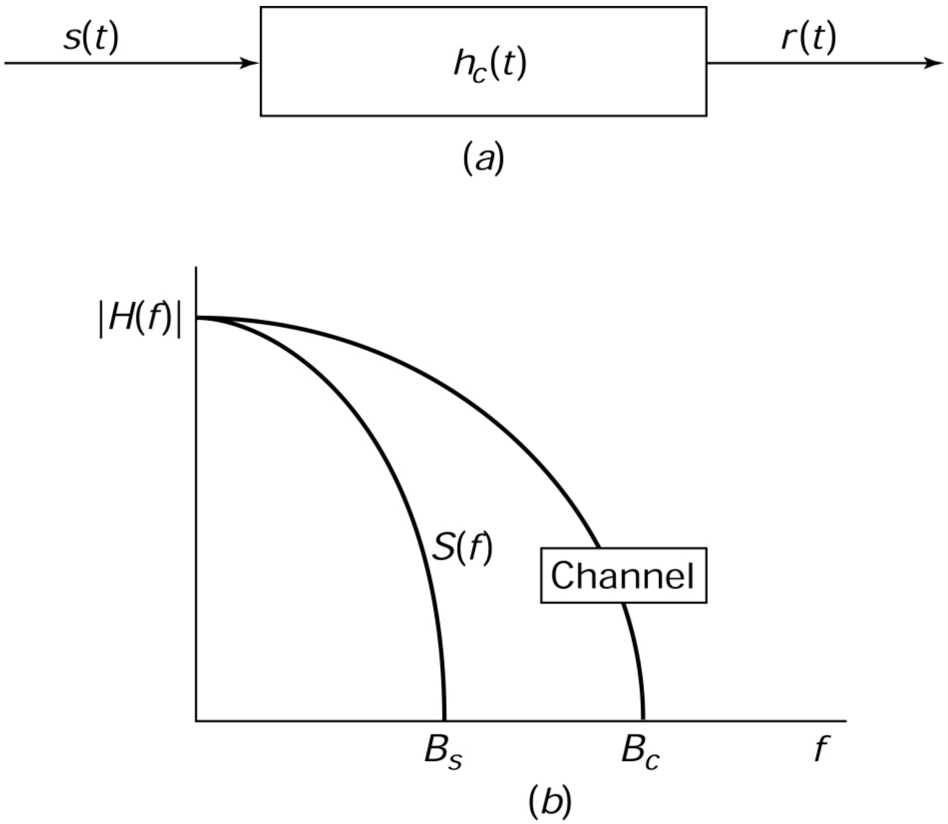




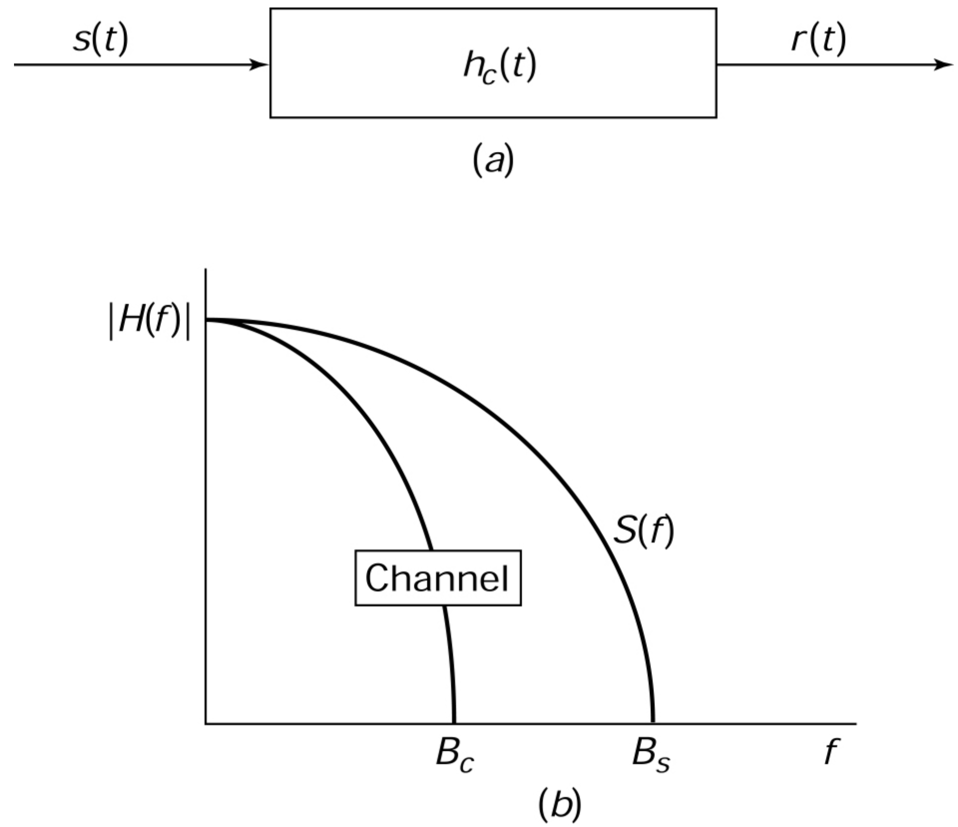




Narrowband and Wideband Signal



Narrowband Signal

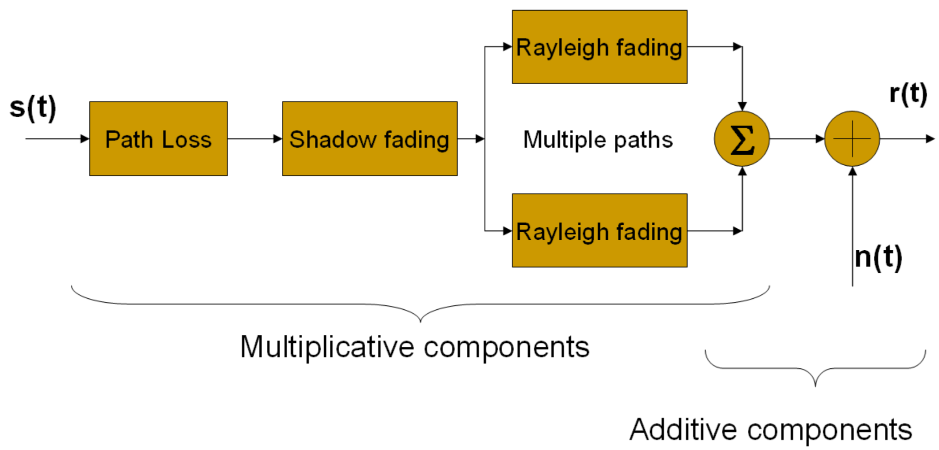


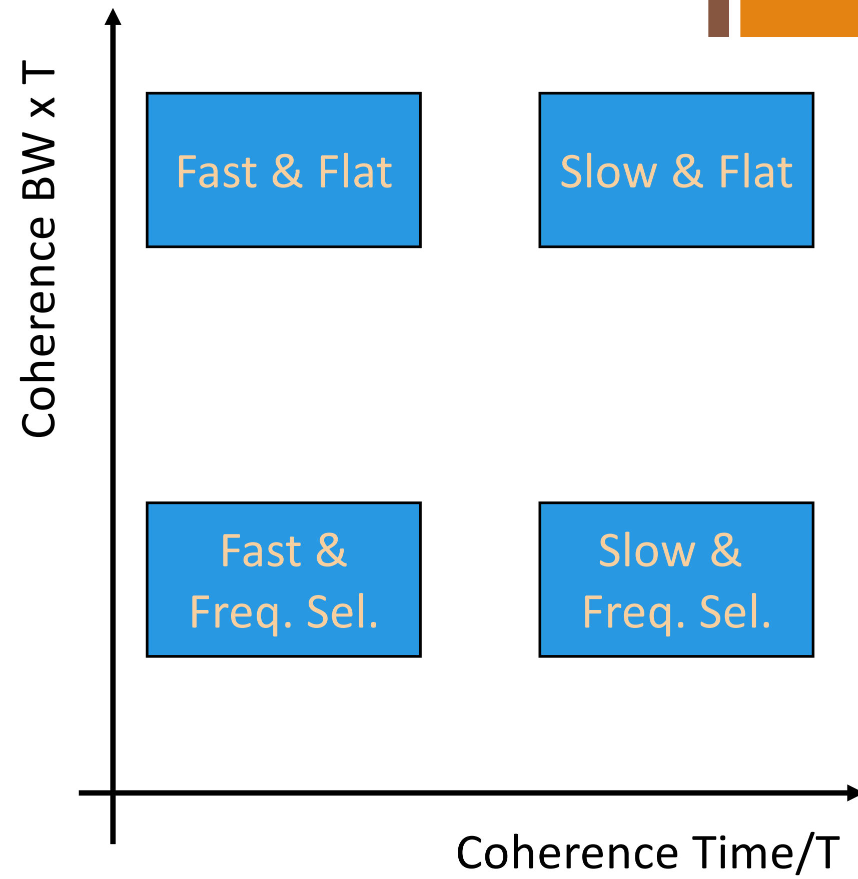
Wideband Signal

Source: Introduction to Wireless Systems by P.M. Shankar, John Wiley & Sons, 2002

Regions of "Influence"

- Depending on the symbol duration (or signal bandwidth) and the channel conditions, we may see different things happening in a radio channel



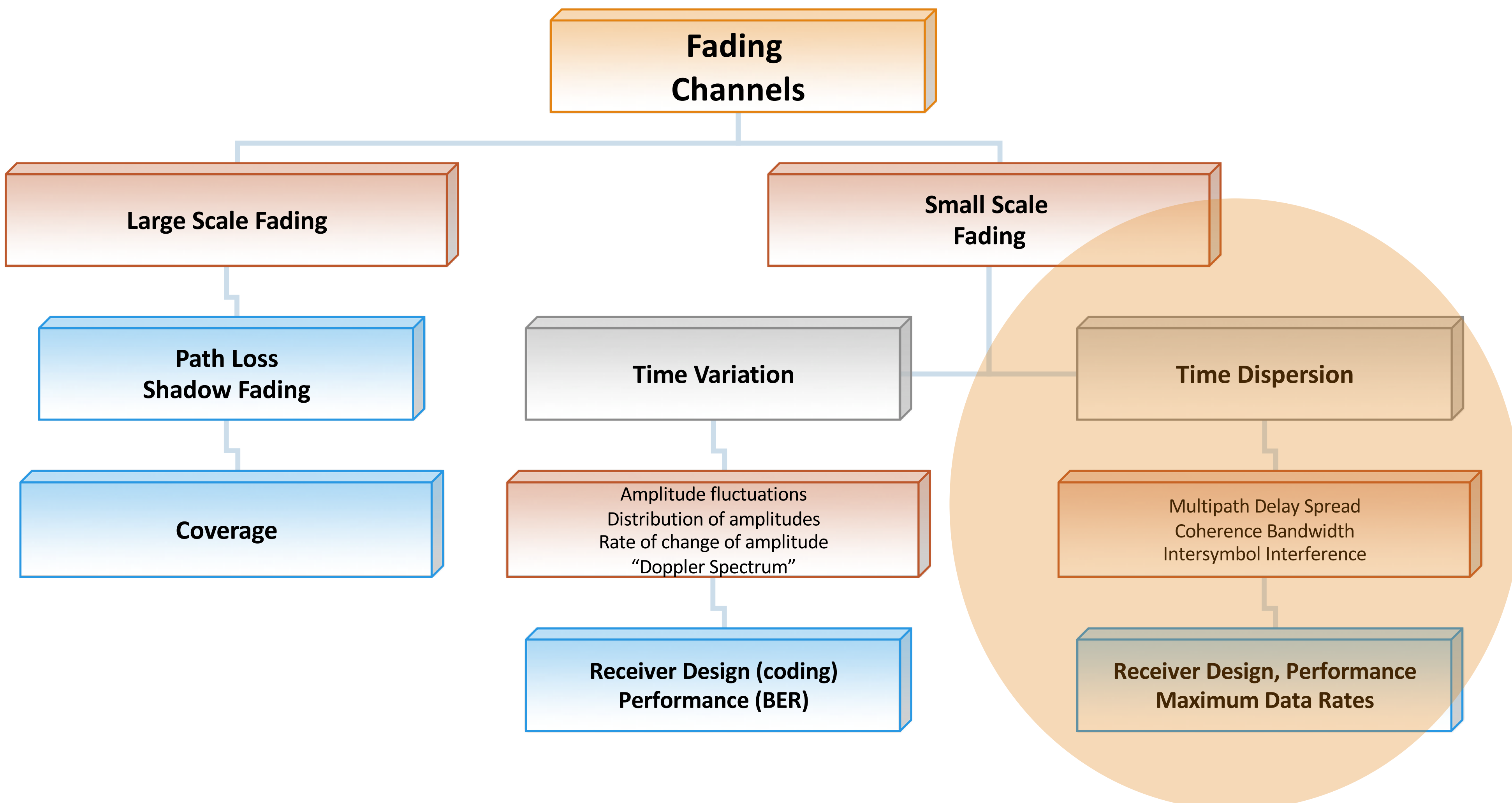


Source: Introduction to Wireless Systems by P.M. Shankar, John Wiley & Sons, 2002

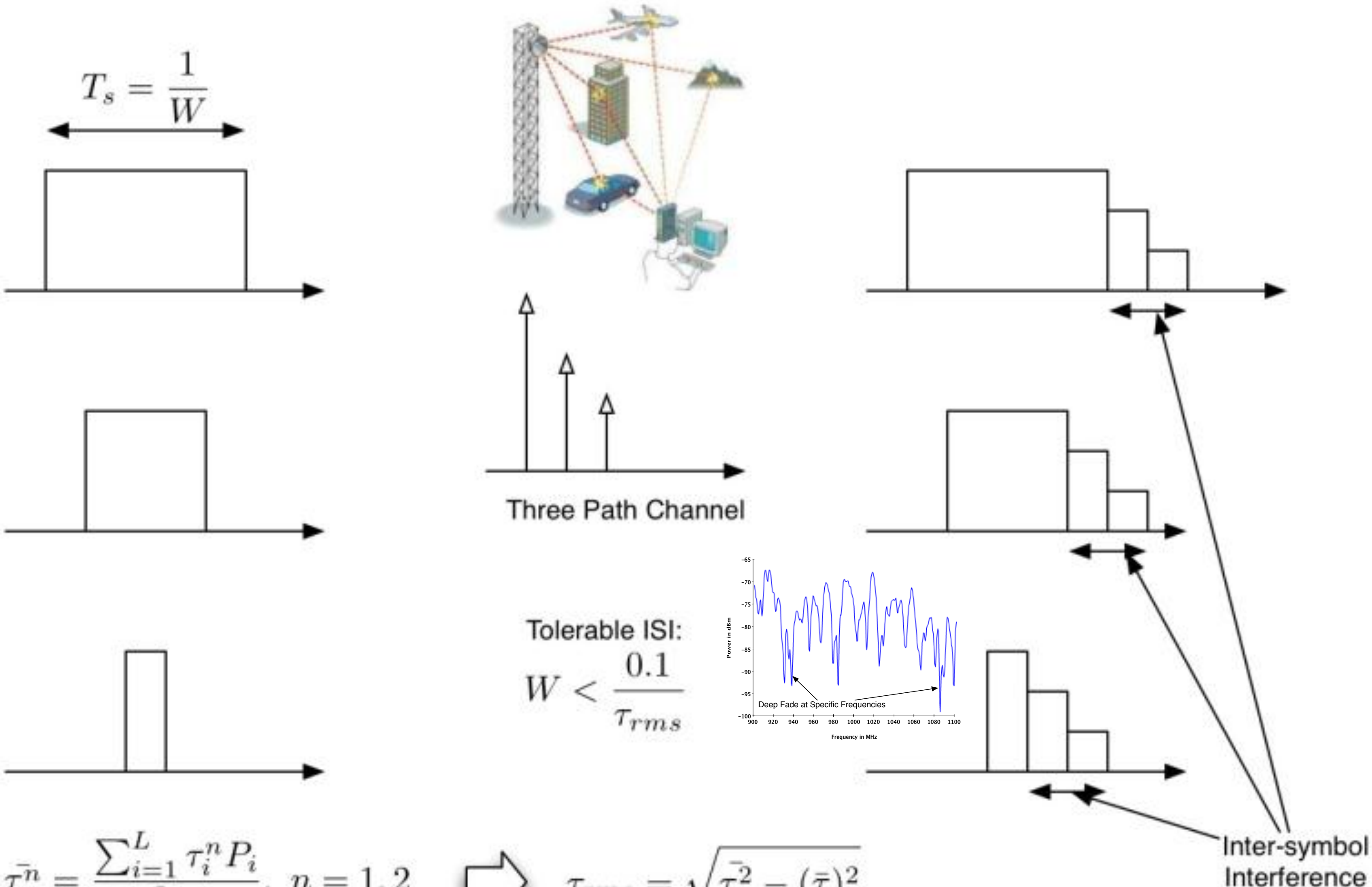
Performance degradation and mitigation

Issue	Performance Affected	Mitigation Technique
Shadow Fading	Coverage	Fade Margin – Increase transmit power or decrease cell size
Time Variation	Bit error rate Packet error rate	Error control coding Interleaving Frequency hopping Diversity
Time Dispersion	Inter-symbol Interference and Irreducible Error Rates	Equalization DS-Spread Spectrum OFDM Directional Antennas

Radio Propagation Characterization



Time Dispersion (Revisited)

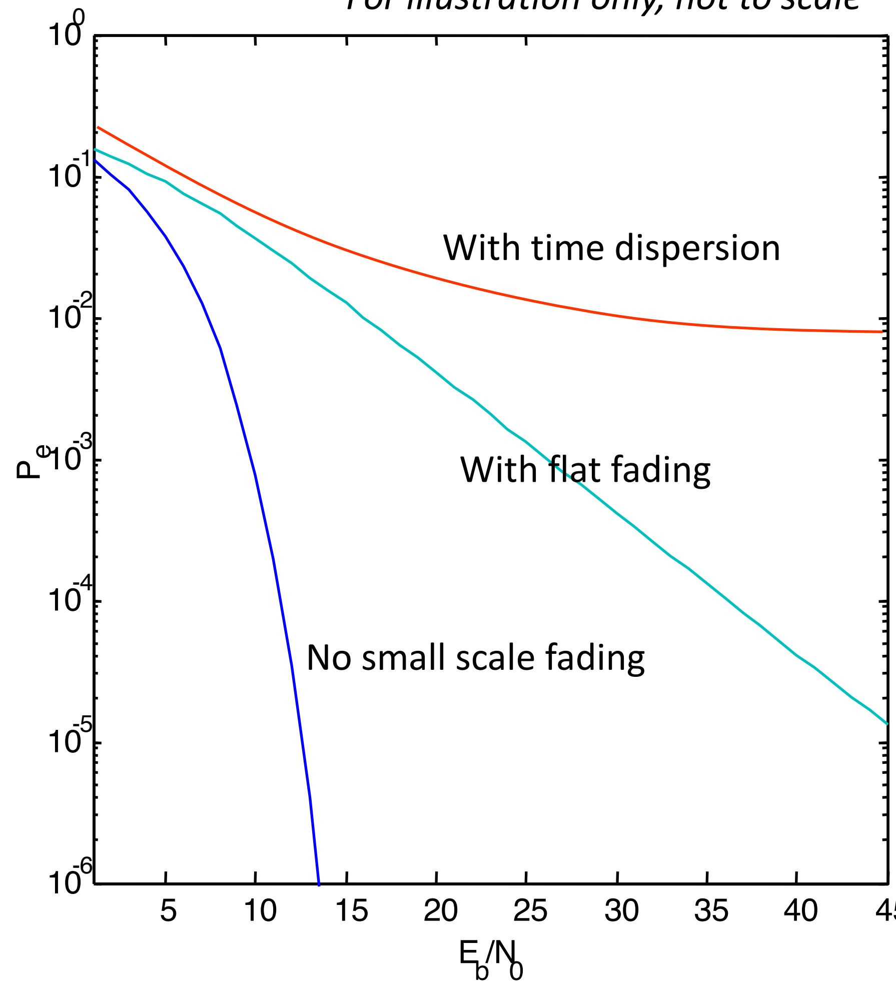


$$\bar{\tau^n} = \frac{\sum_{i=1}^{L} \tau_i^n P_i}{\sum_{i=1}^{L} P_i}, \; n = 1, 2 \quad \Rightarrow \quad \tau_{rms} = \sqrt{\bar{\tau^2} - (\bar{\tau})^2}$$

What does time dispersion do?

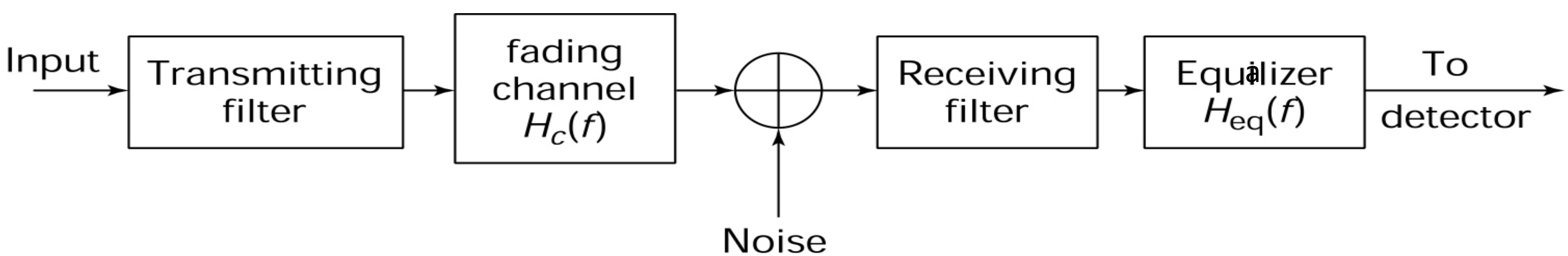
- Multipath dispersion or coherence bandwidth results in **irreducible error rates**
- Even if the power is infinitely increased, there will be large number of errors
- The only means of overcoming the effects of dispersion are to use
  - Equalization
  - Direct sequence spread spectrum
  - Orthogonal frequency division multiplexing

For illustration only, not to scale



Equalization

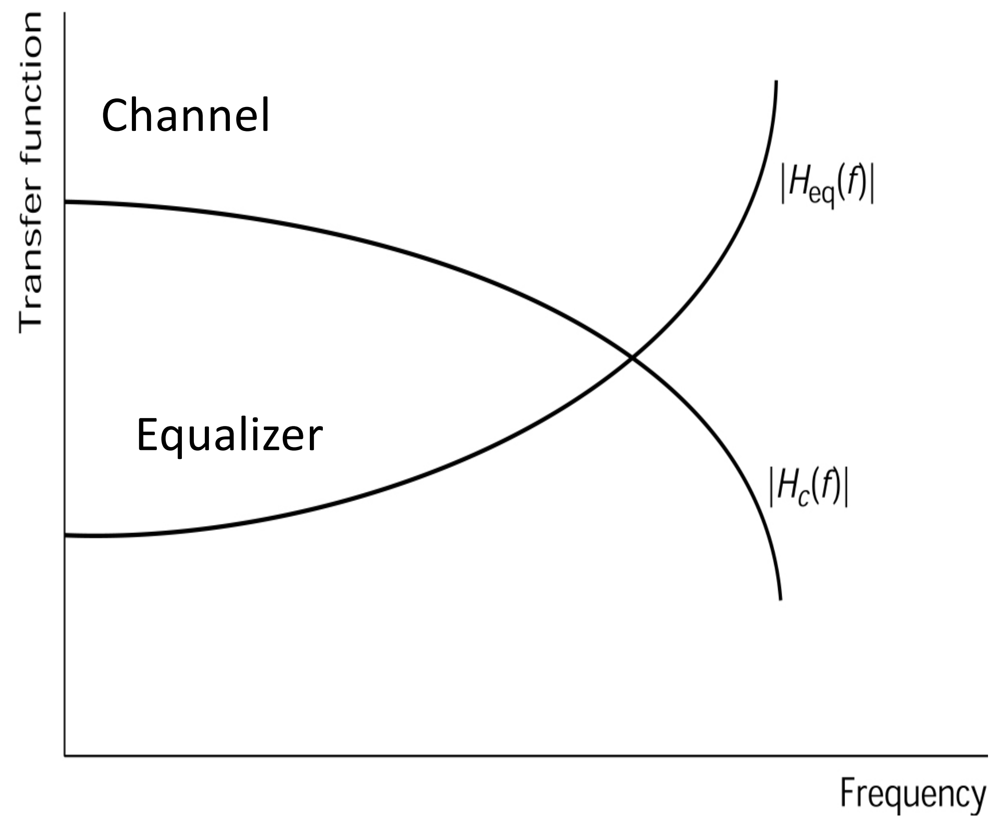
- An equalizer
  - Filter that performs the inverse of the channel
  - Compensate for the distortion created by the frequency selectivity caused by multipath time dispersion
  - Combats ISI
- Equalization
  - Any signal processing that reduces the impact of ISI



Source: Introduction to Wireless Systems by P.M. Shankar, John Wiley & Sons, 2002

Equalization Concepts

- In wireless networks equalizers must be *adaptive*
  - Channel is usually unknown and time varying
  - Equalizers track the time variation and adapt
- Equalizer is usually implemented at baseband



Source: Introduction to Wireless Systems by P.M. Shankar, John Wiley & Sons, 2002

Operating Modes of an Equalizer

- Two step approach to equalization
- Training
  - A known fixed-length sequence is transmitted for the receiver's equalizer to 'train' on
  - This sets the parameters in the equalizer
- Tracking
  - The equalizer tracks the channel changes with the help of the training sequence
  - Uses a channel estimate to compensate for distortions in the unknown sequence

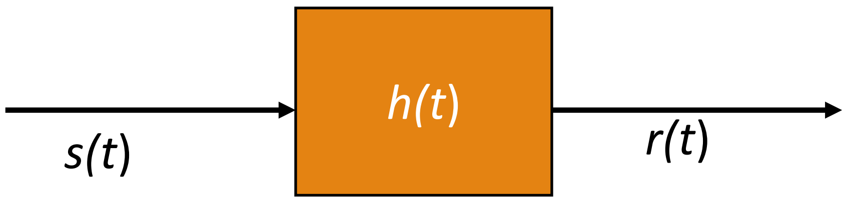
Operating Modes (2)

- Training
  - Training sequence is typically a pseudorandom or fixed binary pattern
  - Needs to be designed to account for the worst case conditions
    - Fastest velocity, largest delay spread, deepest fades
  - Enables the receiver to set its filter coefficients at near optimal values
  - Requires periodic training
    - What is the maximum amount of time you can transmit data before the equalizer has to be trained again?
- Tracking
  - User data is transmitted immediately after training

Operating Modes (3)

- During the training step, the channel response, *h*(*t*) is estimated
- During the tracking step, the input signal, *s*(*t*), is estimated

	Known	Measured	Unknown /Estimated
Training	*s*(*t*)	*r*(*t*)	*h*(*t*)
Tracking	*h*(*t*)	*r*(*t*)	*s*(*t*)



Types of Equalizers

- Linear transversal equalizer, Decision feedback equalizer (DFE), and Maximum likelihood sequence estimator (MLSE)
- Equalizer Algorithms
  - Zero forcing algorithm
    - The equalizer forces the combined channel-equalizer response to be zero at $t = \pm kT$ for all k except one
  - Least mean square (LMS) algorithm
    - Minimizes the mean square error between the equalizer output and desired output
  - Recursive least squares (RLS) algorithm
    - Uses adaptive signal processing and time averages

Comments on Equalization

- Disadvantages of equalizers
  - Complexity & power consumption
  - Numerical errors
- Fractionally spaced equalizers
  - Use taps that are spaced to sample the signal at the Nyquist rate and not the symbol rate
- Equalizers are used in NA-TDMA, GSM and HIPERLAN
  - SC-FDMA used on the LTE uplink can be thought of as frequency domain equalization