pair. Since #26-gauge copper wire has 41 Ω per 1000 feet, a single wire that is one mile long has resistance $= 41 \times 5.28 = 210\ \Omega$. If the detector at B is a 180-Ω resistor, the circuit's net "round-trip" resistance is $210 + 180 + 210 = 600\ \Omega$. Then, a 12-Volt battery at A produces $\frac{12}{600} = 20$ mA. The received signal is $20\ \text{mA} \times 180 = 3.6$ Volts, which is only $\frac{3.6}{12} = 30\%$ of the signal's intensity at A.

While DC signals suffer loss only due to resistance, AC signals also suffer loss due to capacitive and inductive reactance. Even without turns, a wire has a series distributed inductance and a pair of wires has a parallel distributed capacitance. Like its resistance, the total net inductive and capacative reactance of a wire-pair is proportional to the length of the pair. So, the pair's total net opposition to current, its *impedance*, is directly proportional to its length.

If a signal with intensity A_0 is transmitted over a wire, co-axial cable, or fiber, then it loses intensity as it moves along the path. If $A(z)$ is the signal's intensity at position z then the signal loses intensity "exponentially" over distance as:

$$A(z) = A_0 e^{-\alpha z}. \tag{9.3}$$

This equation motivates the use of the "dB scale" in which quantities are expressed logarithmically.

9.2.4 Decibels

While the logarithmic "dB" scale may be initially confusing, it is very useful for handling number that vary over wide ranges. Since gain is multiplicative, translating to a log-scale allows us to add instead of multiplying. If loss is divisive, translating to a log-scale allows us to subtract instead of divide. If loss is exponential over distance, a log scale is convenient because it lets us multiply instead of raising to a power. The telecom industry has universally adopted the *decibel scale*, which:

- Uses the base-10 logarithm, which is then multiplied by 10,
- Measures power instead of voltage or current,
- Measures *relative* power (arguments of logarithms are dimensionless).

Specifically, P_1 is said to be "Y dB greater than" P_2 if:

$$Y\ \text{dB} = 10 \log_{10} \frac{P_1}{P_2}. \tag{9.4}$$

Let P_1 and P_2 be the power intensities, in Watts, of two different signals, such as the input and output powers of an optical transmission link. If P_1 is R times greater than P_2 on the real scale, then P_1 is $10 \log_{10} R$ incrementally bigger than P_2 on the logarithmic dB-scale. Each ten-fold multiplication of power corresponds exactly to a 10 dB increment and each doubling of power corresponds approximately to a 3 dB increment (because $\log_{10} 2 = .3$). So, if a wire transmission line has a loss such that a signal's power is halved every kilometer, we express this as:

$$10 \log_{10} \frac{P_{out}}{P_{in}} = 10 \log_{10}(.5) = -3 \text{ "dB per km."} \tag{9.5}$$

Table 9.1
Some decibel calculations.

P_1/P_2	# dB	P_1/P_2	# dB
1	0	128 (2^7)	21 (3×7)
2	3	160 (16×10)	22 ($12 + 10$)
4 (2^2)	6 (3×2)	200 (2×100)	23 ($3 + 20$)
8 (2^3)	9 (3×3)	256 (2^8)	24 (3×8)
10	10	320 (32×10)	25 ($15 + 10$)
16 (2^4)	12 (3×4)	400 (4×100)	26 ($6 + 20$)
20 (2×10)	13 ($3 + 10$)	512 (2^9)	27 (3×9)
32 (2^5)	15 (3×5)	640 (64×10)	28 ($18 + 10$)
40 (4×10)	16 ($6 + 10$)	800 (8×100)	29 ($9 + 20$)
64 (2^6)	18 (3×6)	1000 (10^3)	30 (10×3)
80 (8×10)	19 ($9 + 10$)	1024 (2^{10})	30 (3×10)
100 (10^2)	20 (10×2)		

Example. Examine these useful numerical examples.

$P_1 : P_2$	dB	
$P_1 = P_2$	$10 \times \log_{10}(1) = 10 \times 0$	$= 0$
$P_1 = 10P_2$	$10 \times \log_{10}(10) = 10 \times 1$	$= 10$
$P_1 = \frac{P_2}{10}$	$10 \times \log_{10}\left(\frac{1}{10}\right) = 10 \times -1$	$= -10$
$P_1 = 10^k P_2$	$10 \times \log_{10}(10^k) = 10 \times k$	$= 10k$
$P_1 = 2P_2$	$10 \times \log_{10}(2) = 10 \times 3.1$	≈ 3
$P_1 = \frac{P_2}{2}$	$10 \times \log_{10}\left(\frac{1}{2}\right) = 10 \times (-3.1)$	≈ -3
$P_1 = 2^k P_2$	$10 \times \log_{10}(2^k) = 10 \times k \times 3.1$	$\approx 3k$

Based on these examples, and their combinations, we can develop Table 9.1 without ever looking up another logarithm value.

Reciprocal ratios just have the negative value of dB. The last two entries in this table are shown to be equal and, of course, they can't be. The problem comes from using $\log_{10}(2) \approx 3$ instead of $\log_{10}(2) = 3.1$. Clearly, the last entry should be 31 (3.1×10) dB, and clearly some of the other entries are also a little inaccurate.

The "dB" scale *always* expresses a signal's power *relative* to the power of some other signal. This comparative relationship is convenient for measuring the gain of an amplifier or the loss in a link. But, the dB-scale can also measure an absolute power by comparing it against a standard value.

While several references are defined, the most common by far is the dBm equation, which is referenced to 1 mW.

$$\text{\# dBm} = 10 \log_{10}\left(\frac{P}{10^{-3}}\right) \tag{9.6}$$

Example. Consider some sample calculations.

Power	dBm
1 μW	−30 dBm
10 μW	−20 dBm
1 mW	0 dBm
3 mW	2 dBm
1 W	30 dBm

Example. Let a signal be transmitted at 20 mW. This transmit power is 20-times (two orders-of-magnitude) greater than 1 mW, or $10 \log \left(\frac{20\text{mW}}{1\text{mW}} \right) = +13$ dBm. If this signal is transmitted down an 11-Km line, whose loss is 3 dB per km, then the power of the received signal is:

$$13 \text{ dBm} - (3 \text{ dB/Km} \times 11 \text{ Km}) = -20 \text{ dBm}, \tag{9.7}$$

which is 20 dB (two orders-of-magnitude) below 1 mW, or 10 μW.

Example. Consider a transmission system: the transmit power is 8 mW, the transmission line attenuates the signal at 5 dB/km, two in-line amplifiers provide gain of 20 each. If the received signal power must be greater than .1 mW, how long may the line be? After converting to dB, the system's equation is $9 \text{ dBm} - 5L \text{ dB} + 2 \times (13 \text{ dB}) = -10$ dBm. So, $L = \frac{(19+26)}{5} = 9$ km.

We have two equations that describe how attenuation reduces signal power along a channel.

- If the channel's exponential attenuation factor is α, then P_T Watts of transmitted signal power attenuates to $P(x) = P_T e^{-\alpha x}$ Watts at a point x meters away from the transmitter.
- In the logarithmic dB-scale, channel attenuation is linear in x. So, if the channel's log-linear attenuation factor is ξ dB/kM, then P_T dBm of transmitted signal power attenuates to $P(x) = P_T - \xi x$ dBm at a point x meters away from the transmitter.

These two attenuation factors are related: $10 \log_{10} \left[\frac{P(x)}{P_T} \right] = 10 \log_{10} \left[e^{-\alpha x} \right] = -10\alpha x$ $\log_{10}[e] = -4.343\alpha x$. The exponential attenuation factor α, with dimension m^{-1}, is used when calculating in Watts and meters; and the log-linear attenuation factor $\xi = 4343\alpha$ specifies the attenuation in dB/kM. So, a channel that is L meters long delivers $P_T e^{-\alpha L}$ Watts, or P_T (in dBm) $-4.343\alpha L$ dBm, of signal power to its receiver.

While the dB scale is defined to translate power ratios, it can be used to translate voltage ratios, and to translate absolute power values. It is common practice to plug the power equation, $P = \frac{V^2}{R}$, into the dB equation to get:

$$\begin{aligned}
\#\ \text{dB} &= 10 \log_{10}\left(\frac{P_1}{P_2}\right) \\
&= 10 \log_{10}\left[\frac{(V_1^2/R)}{(V_2^2/R)}\right] \\
&= 10 \log_{10}\left(\frac{V_1^2}{V_2^2}\right) \\
&= 10 \log_{10}\left[\left(\frac{V_1}{V_2}\right)^2\right] \\
&= 20 \log_{10}\left(\frac{V_1}{V_2}\right) \qquad (9.8)
\end{aligned}$$

So, the dB equation for power ratios seems to extend to voltage ratios, by changing the multiplier from 10 to 20. But, the derivation works only if the resistances associated with each voltage are equal. The second step really should be $10 \log_{10}\left[\frac{(V_1^2/R_1)}{(V_2^2/R_2)}\right]$, and the third step follows only if $R_1 = R_2$, which happens only under a rare coincidence. Nevertheless, this equation is actually used in practice. It shouldn't be. Caveat emptor.

9.3 NOISE

When we communicate acoustically, like when trying to speak while some motor is running, we call the undesired signal "noise." But *electromagnetic interference (EMI)* can add to the electronic analog of a voice signal within a communications system. The primary impediment to *correctly* recovering a signal as it was transmitted is noise. If the noise level is high, we will have unacceptable "crackle" or "static" in analog audio transmission, "snow" in analog television signal transmission, or "bit error rate" in digital transmission.

Noise is itself a signal, and so it can be loosely defined as an *undesirable signal* that impacts the quality of a desired signal. Immediately, we are faced with two questions—what do we mean by the quality of a desired signal and how does noise impact it? The goal of this section is to qualitatively consider physical phenomena that cause noise and use some quantitative methods to (a) model noise manifestations and (b) define signal quality and understand how noise may impact it using these models.

Natural phenomena that cause noise signals that can be voltages, currents or electromagnetic radiation. We can never avoid these natural phenomena because some of them are *internal* to the electronics within a receiver [1]. Other sources of noise are *external* such as man-made interference, ionized atmospheric gases, and solar flares. It is common to model noise as a random process because the natural events that cause noise are either themselves random or too complex to model deterministically. Occasionally, man made transmissions also interfere with one another in which case, it may be possible to consider such noise signals as somewhat deterministic.

We consider some common forms of internal and external noise below.