SADET Module B.2

Homework key

1. Given the following statistics, what is the probability that a woman under the age of 50 that test positive in a mammogram will have cancer:
   1. One percent of women under 50 have breast cancer
   2. Ninety percent of women under 50 who have breast cancer test positive on mammograms
   3. Eight percent of women under 50 will have false positives

P(C)=0.01  
P(~C)=0.99  
P(M|C)=0.9  
P(M|~C)=0.08

P(M) = P(M|C)\*P(C)+P(M|~C)P(~C) = 0.9\*0.01+0.08\*0.99 = 0.0882

P(C|M) = P(M|C)\*P(C)/P(M) = 0.9\*0.01/0.0882 = 0.102

Final probability is 0.102

C: under 50 has cancer

M: under 50 tests positive in mammogram

1. The number of fouls given by an officiating crew in the NBA follows an IID with mean 35 and standard deviation 6. A random sample of 50 games is selected by the commissioner for review. What is the probability that the sample mean of these games is at least 38?

Use central limit theorem for the average number of fouls. Since fouls are IID we expect the sample mean distribution to be normal with mean 35 and standard deviation 6/sqrt(50). Hence, the probability of observing a sample mean greater or equal to 38 is almost 0 (0.0002).

1. The PA Department of Education is suspecting that a specific school has cheated in the SATs. The average SAT score of the 105 students that took the exam in the school under examination is 710. The population average for SAT scores has a mean of 550 and a standard deviation of 70. Can you make any statistical claims on whether the school indeed could have cheated during the exams?

We can set up a hypothesis test:

H0: The school exams come from the overall population

H1: The school exams do not come from the overall population

Under the null assumption, using the CLT, the sample mean of a given sample of SAT scores should follow a normal distribution with mean 550 and standard deviation of 70/sqrt(105). With this in mind the probability of observing an average of 710 for a sample of 105 students is practically 0. Hence, under the assumption that H0 is true the probability of observing this by chance is pretty much 0 and we can reject the null.

Note: this does not mean that the school cheated. It only shows that it is statistically irregular. It can be the case that the school is very good and/or has exceptional students.