IS 2610: Data Structures

Discuss HW 2 problems

Binary Tree (continued)

Introduction to Sorting

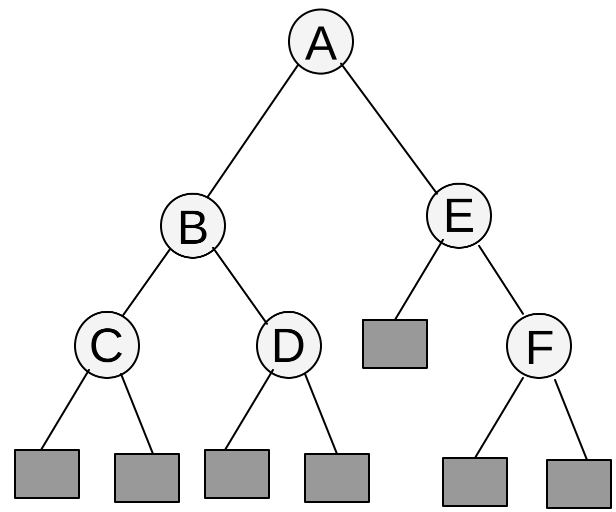
Feb 9, 2004

Binary tree

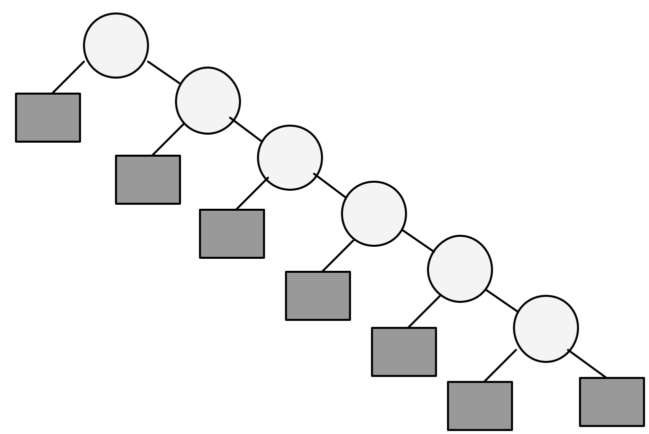
- A binary tree with N internal nodes has 2N links
  - N-1 to internal nodes
    - Each internal node except root has a unique parent
    - Every edge connects to its parent
  - N+1 to external nodes
- Level, height, path
  - Level of a node is 1 + level of parent (Root is at level 0)
  - Height is the maximum of the levels of the tree's nodes
  - Path length is the sum of the levels of all the tree's nodes
  - Internal path length is the sum of the levels of all the internal nodes

Examples

- Level of D ?
- Height of tree?
- Internal length?
- External length?



- Height of tree?
- Internal length?
- External length?



Binary Tree

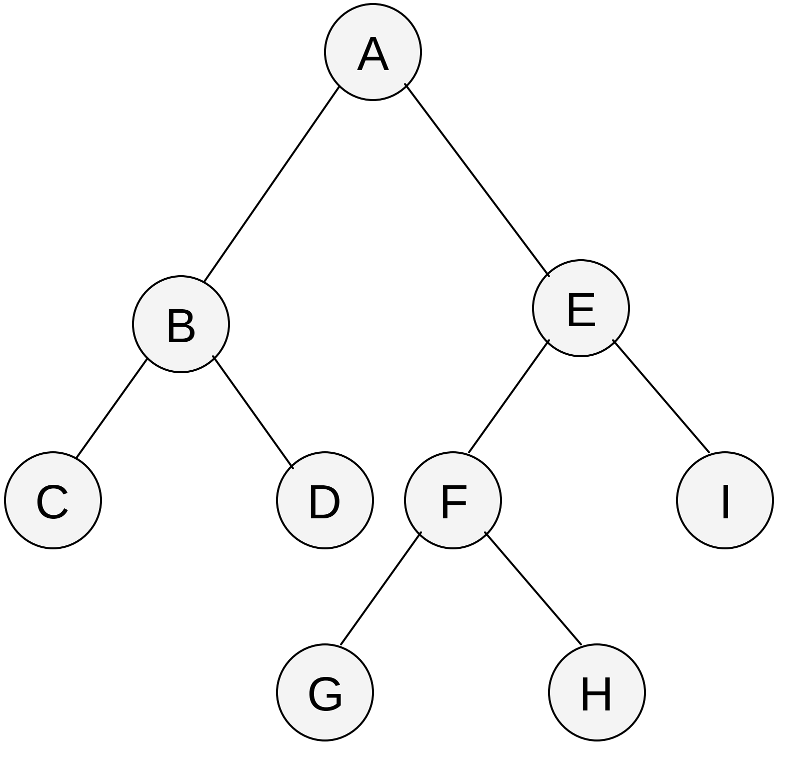
- External path length of any binary tree with N internodes is $2N$ greater than the internal path length
- The height of a binary tree with N internal nodes is at least $\lg N$ and at most N-1
  - Worst case is a degenerate tree: N-1
  - Best case: balanced tree with 2^i nodes at level i.
    - Hence for height: $2^{h-1} < N+1 = 2^h$ – hence h is the height

Binary Tree

- Internal path length of a binary tree with N internal nodes is at least $N\ lg\ (N/4)$ and at most $N(N\text{-}1)/2$
  - Worst case : $N(N\text{-}1)/2$
  - Best case: $(N+1)$ external nodes at height no more than $\lfloor lg\ N \rfloor$
    - $(N+1)\lfloor lg\ N \rfloor - 2N < N \lg (N/4)$

Tree traversal (binary tree)

- Preorder
  - Visit a node,
  - Visit left subtree,
  - Visit right subtree
- Inorder
  - Visit left subtree,
  - Visit a node,
  - Visit right subtree
- Postorder
  - Visit left subtree,
  - Visit right subtree
  - Visit a node



Recursive / Nonrecursive Preorder

```
void traverse(link h, void (*visit)(link))
{
   If (h == NULL) return;
   (*visit)(h);
   traverse(h->l, visit);
   traverse(h->r, visit);
}
```

```
void traverse(link h, void (*visit)(link))
{
   STACKinit(max);
   STACKpush(h);
  while (!STACKempty())
  {
        (*visit)(h = STACKpop());
        if (h->r != NULL) STACKpush(h->r);
       if (h->l != NULL) STACKpush(h->l);
  }
}
```

Recursive binary tree algorithms

- Exercise on recursive algorithms:
  - Counting nodes
  - Finding height

Sorting Algorithms: Selection sort

- Basic idea
  - Find smallest element and put in the first place
  - Find next smallest and put in second place
  - ..
- Try for

  2 6 3 1 5

```
#define key(A) (A)
#define less(A, B) (key(A) < key(B))
#define exch(A, B) { Item t = A; A = B; B = t; }

void selection (Item a[], int l, int r)
{
   int i, j;
   for (i = l; i < r; i++)
       { int min = i;
             for (i = i+1; i <= r; j++)   {
                  if (less(a[j], a[min]) min = j;

             exch(a[i], a[min]);
       }
}
```

Sorting Algorithms: Selection sort

- Recursive?
  - Eliminate the outer loop
  - Find the minimum and place it in the first place
  - Make recursive call to sort the remaining parts of the array
- Complexity
  - i goes from 1 to N-1
  - There is one exchange per iteration; total N-1
  - There is N-i comparisons per iteration
    - $(N-1) + (N+2) + .. + 2 + 1 = N(N-1)/2$

Sorting Algorithms: Bubble sort

- Bubble sort
  - Move through the elements exchanging adjacent pairs if the first one is larger than the second
  - Try out !
    - 2 6 3 1 5

```
#define key(A) (A)
#define less(A, B) (key(A) < key(B))
#define exch(A, B) { Item t = A; A = B; B = t; }

void selection (Item a[], int l, int r)
{
   int i, j;
   for (i = l; i < r; i++)
       for (j = r; j > r; j--)
           if (less(a[j-1], a[j]) exch(a[j-1], a[j]);
}
```

Sorting Algorithms: Bubble sort

- Recursive?
- Complexity
  - i^{th} pass through the loop – $(N-i)$ compare-and-exchange
  - Hence $N(N-1)/2$ compare-and-exchange is the worst case
  - What would be the minimum number of exchanges?

Sorting Algorithms: Insertion sort

- Insertion sort
  - “People” method

  2 6 3 1 5
- Complexity
  - About $N^2/4$
  - *About half of the left array*

```
#define less(A, B) (key(A) < key(B))
#define exch(A, B) { Item t = A; A = B; B = t; }
#define compexch(A, B) if (less(B, A)) exch(A, B)

void insertion(Item a[], int l, int r) {
   int i;
   for (i = r; i > l ; i--) compexch(a[i-1], a[i]);
   for (i = l+2; i <= r; i++)   {
      int j = i; Item v = a[i];
      while (less(v, a[j-1])) {
          a[j] = a[j-1]; j--;
      }
      a[j] = v;
   }
}
```

Sorting Algorithms: Shell sort

- Extend of insertion sort
  - Taking every h^{th} element will
  - Issues
    - Increment sequence?

h = 13

ASORTINGEXAMPLE

ASORTINGEXAMPLE

ASORTINGEXAMPLE

AEORTINGEXAMPLS

```
void shellsort(Item a[], int l, int r)
  { int i, j, h;
    for (h = 1; h <= (r-l); h = 3*h+1) ;
    for ( ; h > 0;)
      h = h/3;
      for (i = l+h; i <= r; i++)
        { int j = i; Item v = a[i];
          while (j >= l+h && less(v, a[j-h]))
            { a[j] = a[j-h]; j -= h; }
          a[j] = v;
        }
  }
```

Sorting Algorithms: Shell sort

h = 4

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…

```
void shellsort(Item a[], int l, int r)
  { int i, j, h;
    for (h = 1; h <= (r-l); h = 3*h+1) ;
    for ( ; h > 0;)
      h = h/3;
      for (i = l+h; i <= r; i++)
        { int j = i; Item v = a[i];
          while (j >= l+h && less(v, a[j-h]))
            { a[j] = a[j-h]; j -= h; }
          a[j] = v;
        }
  }
```

Sorting Algorithms: Quick sort

- A divide and conquer algorithm
  - Partition an array into two parts
  - Sort the parts independently
  - Crux of the method is the partitioning process
    - Arranges the array to make the following three conditions hold
      - The element a[i] is in the final place in the array for some I
      - None of the elements in a[l]….a[i-1] is greater than a[i]
      - None of the elements in a[i+1]….a[r] is less than a[i]

Sorting Algorithms: Quick sort

```
 int partition(Item a[], int l, int r);
void quicksort(Item a[], int l, int r)
  { int i;
    if (r <= l) return;
    i = partition(a, l, r);
    quicksort(a, l, i-1);
    quicksort(a, i+1, r);
  }
```

```
int partition(Item a[], int l, int r)
 { int i = l-1, j = r; Item v = a[r];
   for (;;)
     {
       while (less(a[++i], v)) ;
       while (less(v, a[--j])) if (j == l) break;
       if (i >= j) break;
       exch(a[i], a[j]);
     }
   exch(a[i], a[r]);
   return i;
 }
```

Sorting Algorithms: Quick sort

7 20 5 1 3 11 8 10 2 9

7 20 5 1 3 11 8 10 2 9

7 2 5 1 3 11 8 10 20 9

7 2 5 1 3 11 8 10 20 9

7 2 5 1 3 8 11 10 20 9

i *j*

7 2 5 1 3 8 9 10 20 11

i

```
int partition(Item a[], int l, int r)
 { int i = l-1, j = r; Item v = a[r];
   for (;;)
     {
       while (less(a[++i], v)) ;
       while (less(v, a[--j])) if (j == l) break;
       if (i >= j) break;
       exch(a[i], a[j]);
     }
   exch(a[i], a[r]);
   return i;
 }
```