# IS 2150 / TEL 2810 Introduction to Security



James Joshi Assistant Professor, SIS

> Lecture 4 September 20, 2007

Access Control Model Foundational Results



## Back to .. Access Control Matrix



## **Protection System**

- State of a system
  - Current values of
    - memory locations, registers, secondary storage, etc.
    - other system components
- Protection state (P)
  - A system state that is considered secure
- A protection system
  - Captures the conditions for state transition
  - Consists of two parts:
    - A set of generic rights
    - A set of commands



### **Protection System**

- Subject (S: set of all subjects)
  - Eg.: users, processes, agents, etc.
- Object (O: set of all objects)
  - Eg.:Processes, files, devices
- Right (R: set of all rights)
  - An action/operation that a subject is allowed/disallowed on objects
  - Access Matrix A:  $a[s, o] \subseteq R$
- Set of Protection States: (S, O, A)
  - Initial state  $X_0 = (S_{0'}, O_{0'}, A_0)$

### **State Transitions**

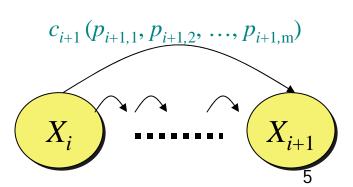
 $X_i \vdash \tau_{i+1} X_{i+1}$ : upon transition  $\tau_{i+1}$ , the system moves from state  $X_i$  to  $X_{i+1}$ 

 $X \vdash^* Y$ : the system moves from state X to Y after a set of transitions

\*
X
Y

 $X_i$ 

 $X_i \vdash c_{i+1} (p_{i+1,1}, p_{i+1,2}, ..., p_{i+1,m}) X_{i+1}$ : state transition upon a command For every command there is a sequence of state transition operations





## Primitive commands (HRU)

Create subject s	Creates new row, column in ACM; s does not exist prior to this	
Create object o	Creates new column in ACM o does not exist prior to this	
Enter $r$ into $a[s, o]$	Adds $r$ right for subject $s$ over object $o$ Ineffective if $r$ is already there	
Delete $r$ from $a[s, o]$	Removes $r$ right from subject $s$ over object $o$	
Destroy subject s	Deletes row, column from ACM;	
Destroy object o	Deletes column from ACM	



## Primitive commands (HRU)

Create subject s

Creates new row, column in ACM; s does not exist prior to this

Precondition:  $s \notin S$ 

Postconditions:

$$S' = S \cup \{ s \}, O' = O \cup \{ s \}$$

$$(\forall y \in O')[a'[s, y] = \emptyset]$$
 (row entries for s)

$$(\forall x \in S')[a'[x, s] = \emptyset]$$
 (column entries for s)

$$(\forall x \in S)(\forall y \in O)[a^{\cdot}[x, y] = a[x, y]]$$



## Primitive commands (HRU)

Enter r into a[s, o]

Adds r right for subject s over object o Ineffective if r is already there

Precondition:  $s \in S$ ,  $o \in O$ 

Postconditions:

$$S' = S_i O' = O$$

$$a'[s, o] = a[s, o] \cup \{r\}$$

$$(\forall x \in S')(\forall y \in O')$$

$$[(x, y) \neq (s, o) \rightarrow a'[x, y] = a[x, y]]$$



## System commands

[Unix] process p creates file f with owner read and write (r, w) will be represented by the following:

```
Command create\_file(p, f)
Create object f
Enter own into a[p,f]
Enter r into a[p,f]
Enter w into a[p,f]
End
```



## System commands

Process p creates a new process q

```
Command spawn\_process(p, q)

Create subject q;

Enter own into a[p,q]

Enter r into a[p,q]

Enter w into a[p,q]

Enter r into a[q,r]

Enter w into a[q,r]

Parent and child can signal each other

End
```



## System commands

 Defined commands can be used to update ACM

```
Command make\_owner(p, f)
Enter own into a[p,f]
End
```

- Mono-operational:
  - the command invokes only one primitive



### **Conditional Commands**

Mono-operational + monoconditional

```
Command grant_read_file(p, f, q)

If own in a[p,f]

Then

Enter r into a[q,f]

End
```



### **Conditional Commands**

Mono-operational + biconditional

```
Command grant_read_file(p, f, q)

If r in a[p,f] and c in a[p,f]

Then

Enter r into a[q,f]

End
```

Why not "OR"??



## Fundamental questions

- How can we determine that a system is secure?
  - Need to define what we mean by a system being "secure"
- Is there a generic algorithm that allows us to determine whether a computer system is secure?



## What is a secure system?

- A simple definition
  - A secure system doesn't allow violations of a security policy
- Alternative view: based on distribution of rights to the subjects
  - Leakage of rights: (unsafe with respect to right r)
    - Assume that A representing a secure state does not contain a right r in any element of A.
    - A right r is said to be leaked, if a sequence of operations/commands adds r to an element of A, which did not contain r



## What is a secure system?

- Safety of a system with initial protection state  $X_o$ 
  - Safe with respect to r: System is safe with respect to r if r can never be leaked
  - Else it is called unsafe with respect to right

## Safety Problem: formally



- initial state  $X_O = (S_O, O_O, A_O)$
- Set of primitive commands c
- r is not in  $A_0[s, o]$
- Can we reach a state  $X_n$  where
  - $\exists s,o$  such that  $A_n[s,o]$  includes a right r not in  $A_o[s,o]$ ?
    - If so, the system is not safe
    - But is "safe" secure?



#### Undecidable Problems

#### Decidable Problem

- A decision problem can be solved by an algorithm that halts on all inputs in a finite number of steps.
- Undecidable Problem
  - A problem that cannot be solved for all cases by any algorithm whatsoever



# Decidability Results (Harrison, Ruzzo, Ullman)

#### Theorem:

• Given a system where each command consists of a single *primitive* command (mono-operational), there exists an algorithm that will determine if a protection system with initial state X<sub>0</sub> is safe with respect to right r.



- Proof: determine minimum commands k to leak
  - Delete/destroy: Can't leak (or be detected)
  - Create/enter: new subjects/objects "equal", so treat all new subjects as one
    - No test for absence
    - Tests on A[s<sub>1</sub>, o<sub>1</sub>] and A[s<sub>2</sub>, o<sub>2</sub>] have same result as the same tests on A[s<sub>1</sub>, o<sub>1</sub>] and A[s<sub>1</sub>, o<sub>2</sub>] = A[s<sub>1</sub>, o<sub>2</sub>]  $\cup$  A[s<sub>2</sub>, o<sub>2</sub>]
  - If *n* rights leak possible, must be able to leak k= $n(|S_0|+1)(|O_0|+1)+1$  commands
  - Enumerate all possible states to decide



## Decidability Results (Harrison, Ruzzo, Ullman)

- It is undecidable if a given state of a given protection system is safe for a given generic right
- For proof need to know Turing machines and halting problem



#### The halting problem:

 Given a description of an algorithm and a description of its initial arguments, determine whether the algorithm, when executed with these arguments, ever halts (the alternative is that it runs forever without halting).

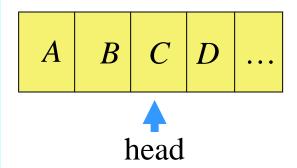


- Theorem: It is undecidable if a given state of a given protection system is safe for a given generic right
- Reduce TM to Safety problem
  - If Safety problem is decidable then it implies that TM halts (for all inputs) – showing that the halting problem is decidable (contradiction)
- TM is an abstract model of computer
  - Alan Turing in 1936



## Turing Machine

- TM consists of
  - A tape divided into cells; infinite in one direction
  - A set of tape symbols M
    - M contains a special blank symbol b
  - A set of states K
  - A head that can read and write symbols
  - An action table that tells the machine how to transition
    - What symbol to write
    - How to move the head ('L' for left and 'R' for right)
    - What is the next state

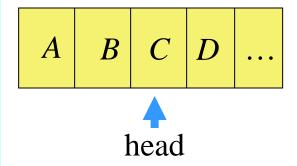


Current state is *k*Current symbol is *C* 



## Turing Machine

- Transition function  $\delta(k, m) = (k', m', L)$ :
  - in state k, symbol m on tape location is replaced by symbol m',
  - head moves to left one square, and TM enters state k'
- Halting state is  $q_f$ 
  - TM halts when it enters this state

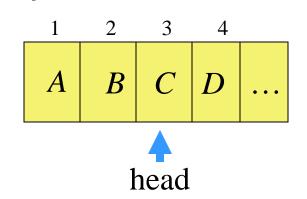


Current state is *k*Current symbol is *C* 

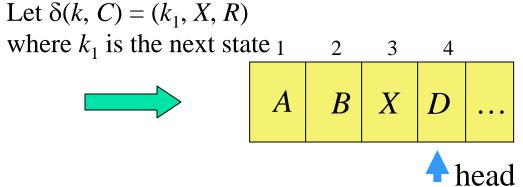
Let  $\delta(k, C) = (k_1, X, R)$ where  $k_1$  is the next state

## 4

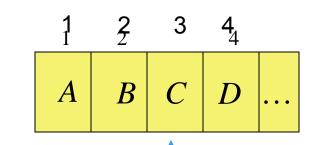
## Turing Machine



Current state is *k*Current symbol is *C* 







Current state is *k* 



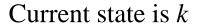
Current symbol is *C* 

Proof: Reduce TM to safety problem

- Symbols, States ⇒ rights
- Tape cell ⇒ subject
- Cell  $s_i$  has  $A \Rightarrow s_i$  has A rights on itself
- Cell  $s_k \Rightarrow s_k$  has end rights on itself
- State p, head at  $s_i \Rightarrow s_i$  has p rights on itself
- Distinguished Right own:
  - $S_i$  owns  $S_i + 1$  for  $1 \le i < k$

	$s_1$	$s_2$	$s_3$	<i>s</i> <sub>4</sub>	
$s_1$	A	own			
$s_2$		В	own		
$s_3$			C k	own	
$S_4$				D end	

## Command Mapping (Left move)





Current symbol is *C* 

$$\delta(k, \mathbf{C}) = (k_1, \mathbf{X}, \mathbf{L})$$

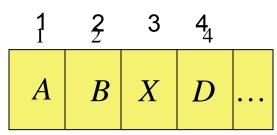
$$\delta(k, C) = (k_1, X, L)$$

#### If head is not in leftmost

command 
$$c_{k,C}(s_i, s_{i-1})$$
  
if own in  $a[s_{i-1}, s_i]$  and  $k$  in  $a[s_i, s_i]$   
and  $C$  in  $a[s_i, s_i]$   
then  
delete  $k$  from  $A[s_i, s_i]$ ;  
delete  $C$  from  $A[s_i, s_i]$ ;  
enter  $X$  into  $A[s_i, s_i]$ ;  
enter  $k_1$  into  $A[s_{i-1}, s_{i-1}]$ ;

	$s_1$	$s_2$	$s_3$	$S_4$	
$s_1$	A	own			
$s_2$		В	own		
$s_3$			C k	own	
$s_4$				D end	

## Command Mapping (Left move)



Current state is  $k_1$ 



Current symbol is *D* head

$$\delta(k, \mathbf{C}) = (k_1, \mathbf{X}, \mathbf{L})$$

$$\delta(k, C) = (k_1, X, L)$$

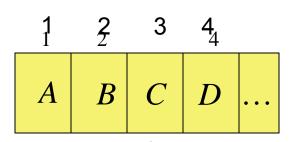
#### If head is not in leftmost

command 
$$c_{k,C}(s_i, s_{i-1})$$
  
if own in  $a[s_{i-1}, s_i]$  and  $k$  in  $a[s_i, s_i]$   
and  $C$  in  $a[s_i, s_i]$   
then  
delete  $k$  from  $A[s_i, s_i]$ ;  
delete  $C$  from  $A[s_i, s_i]$ ;  
enter  $X$  into  $A[s_i, s_i]$ ;  
enter  $k_1$  into  $A[s_{i-1}, s_{i-1}]$ ;

If head is in leftmost both  $s_i$ ,  $s_{i-1}$  are  $s_1$ 

	$s_1$	$s_2$	$s_3$	$S_4$	
$s_1$	A	own			
$s_2$		$\mathbf{B} k_1$	own		
$s_3$			X	own	
$S_4$				D end	

# Command Mapping (Right move)



Current state is *k* 



Current symbol is *C* 

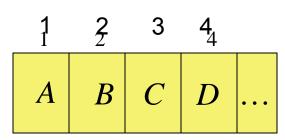
$$\delta(k, C) = (k_1, X, R)$$

$$\delta(k, \mathbf{C}) = (k_1, \mathbf{X}, \mathbf{R})$$

command $c_{k,C}(s_i, s_{i+1})$
if own in $a[s_i, s_{i+1}]$ and k in
$a[s_i, s_i]$ and $C$ in $a[s_i, s_i]$
then
<b>delete</b> $k$ <b>from</b> $A[s_i,s_i]$ ;
<b>delete</b> C <b>from</b> $A[s_i, s_i]$ ;
enter X into $A[s_i,s_i]$ ;
<b>enter</b> $k_1$ <b>into</b> $A[s_{i+1}, s_{i+1}];$
end

	$s_1$	$s_2$	$s_3$	$S_4$	
$s_1$	A	own			
$s_2$		В	own		
$s_3$			C k	own	
$s_4$				D end	

# Command Mapping (Right move)



Current state is  $k_1$ 



Current symbol is *C* 

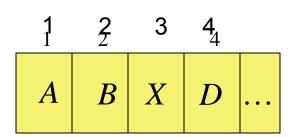
$$\delta(k, C) = (k_1, X, R)$$

$$\delta(k, \mathbf{C}) = (k_1, \mathbf{X}, \mathbf{R})$$

command $c_{k,C}(s_i, s_{i+1})$
if own in $a s_i, s_{i+1} $ and k in
$a[s_i, s_i]$ and $C$ in $a[s_i, s_i]$
then
<b>delete</b> $k$ <b>from</b> $A[s_i,s_i]$ ;
delete C from $A[s_i, s_i]$ ;
enter X into $A[s_i, s_i]$ ;
enter $k_1$ into $A[s_{i+1}, s_{i+1}];$
end

$s_1$	$s_2$	$s_3$	$S_4$	
A	own			
	В	own		
		X	own	
			$D k_1$ end	
		A own	A own B own	A own B own X own





Current state is  $k_1$ 



Current symbol is *C* 

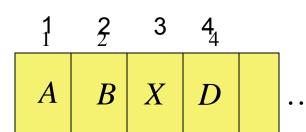
$$\delta(k_1, D) = (k_2, Y, R)$$
 at end becomes

$$\delta(k_1, \mathbf{C}) = (k_2, \mathbf{Y}, \mathbf{R})$$

<b>command</b> crightmost <sub>k,C</sub> ( $s_i$ , $s_{i+1}$ ) <b>if</b> end <b>in</b> $a[s_i$ , $s_i$ ] <b>and</b> $k_1$ <b>in</b> $a[s_i$ , $s_i$ ] <b>and</b> D
in $a[s_i,s_i]$
then
<b>delete</b> end <b>from</b> $a[s_i,s_i]$ ;
create subject $s_{i+1}$ ;
enter own into $a[s_i, s_{i+1}];$
enter end into $a[s_{i+1}, s_{i+1}];$
<b>delete</b> $k_1$ <b>from</b> $a[s_i,s_i];$
<b>delete</b> D <b>from</b> $a[s_i,s_i];$
enter Y into $a[s_i, s_i]$ ;
enter $k_2$ into $A[s_i,s_i]$ ;
end

$s_3$	$S_4$	
$\imath$		
own		
X	own	
	$D k_1$ end	
	own	own X own





Current state is  $k_1$ 



Current symbol is *D* 

$$\delta(k_1, D) = (k_2, Y, R)$$
 at end becomes

$$\delta(k_1, D) = (k_2, Y, R)$$

<b>command</b> crightmost <sub>k,C</sub> ( $s_i$ , $s_{i+1}$ ) <b>if</b> end <b>in</b> $a[s_i$ , $s_i]$ <b>and</b> $k_1$ <b>in</b> $a[s_i$ , $s_i]$ <b>and</b> D
if end in $a[s_i,s_i]$ and $k_1$ in $a[s_i,s_i]$ and D
in $a[s_i,s_i]$
then
<b>delete</b> end from $a[s_i,s_i]$ ;
create subject $s_{i+1}$ ;
enter own into $a[s_i, s_{i+1}];$
enter end into $a[s_{i+1}, s_{i+1}];$
<b>delete</b> $k_1$ <b>from</b> $a[s_i,s_i]$ ;
<b>delete</b> D <b>from</b> $a[s_i, s_i]$ ;
enter Y into $a[s_i, s_i]$ ;
enter $k_2$ into $A[s_i,s_i]$ ;
end

	$s_1$	$s_2$	$s_3$	$S_4$	$s_5$
$s_1$	A	own			
$s_2$		В	own		
$s_3$			X	own	
$S_4$				Y	own
<i>S</i> <sub>5</sub>					$b k_1$ end



### Rest of Proof

- Protection system exactly simulates a TM
  - Exactly 1 end right in ACM
  - Only 1 right corresponds to a state
  - Thus, at most 1 applicable command in each configuration of the TM
- If TM enters state  $q_f$ , then right has leaked
- If safety question decidable, then represent TM as above and determine if  $q_f$  leaks
  - Leaks halting state ⇒ halting state in the matrix ⇒ Halting state reached
- Conclusion: safety question undecidable



### Other results

- For protection system without the create primitives, (i.e., delete create primitive); the safety question is complete in P-SPACE
- It is undecidable whether a given configuration of a given monotonic protection system is safe for a given generic right
  - Delete destroy, delete primitives;
  - The system becomes monotonic as they only increase in size and complexity
- The safety question for biconditional monotonic protection systems is undecidable
- The safety question for monoconditional, monotonic protection systems is decidable
- The safety question for monoconditional protection systems with create, enter, delete (and no destroy) is decidable.
- Observations
  - Safety is undecidable for the generic case
  - Safety becomes decidable when restrictions are applied