IS2150/TEL2810 Introduction to Security

Homework 4

Total Points: 50

Due Date: September Oct 5, 2007

1) From Section 4.8 Do exercise 1, 5 **[15 Points]**

2) From Section 5.5 Do exercise 1, 2 **[15 Points]**

3) Exercise on Lattice **[20 Points]**

Consider set of digits $D = \{1, 2, 3\}$. Let S be set of all numbers containing two digits from D; i.e., each element $a \in S$ can be written as $a = a_1a_2$ where $a_1, a_2 \in D$ (i.e., they are elements of D). For instance $a = a_1a_2 = 12$ is an element of S, as $a_1 = 1$ and $a_2 = 2$. Let relation $\precsim$ be the "*dominance*" relation on S. For every $a, b \in S$ we say a is *dominated* by b (written as $a \precsim b$) if and only if $a_1 \leq b_1$ and $a_2 \leq b_2$. (here $\leq$ is the "*less than or equal to*" relation on natural numbers 1, 2, and 3, i.e., $1 \leq 2$, $2 \leq 3$, $1 \leq 1$, etc.)

1. Does relation $\precsim$ generate a *partial order* or a *total order* on the elements of D? Draw the *Hasse* diagram for the order it generates.

2. Answer the following and justify it.
    a. Does S and $\precsim$ form a lattice?
    b. Now remove elements 21 and 22 from set S, i.e. Does the resulting set and $\precsim$ form a lattice?