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The Concept of a Z-number—a New Direction in Uncertain Computation

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Abstract

Decisions are based on information. To be useful, information must be reliable. Basically, the concept of a Z-number relates to the issue of reliability of information. A Z-number, Z, has two components, Z=(A,B). The first component, A, is a restriction (generalized constraint) on the values which a real-valued uncertain variable, X, is allowed to take. The second component, B, is a measure of reliability (certainty) of the first component. Typically, A and B are described in a natural language. Example: (about 45 minutes, very sure). An important issue relates to computation with Z-numbers. Examples: What is the sum of (about 45 minutes, very sure) and (about 30 minutes, sure)? What is the square root of (approximately 100, likely)? Computation with Z-numbers falls within the province of Computing with Words (CW or CWW).

To view the concept of a Z-number in a general perspective it is helpful to construct a conceptual framework in which there are levels of generality of uncertain computation, with each level representing a class of restrictions. The lowest level, referred to as the ground level, is the space of real numbers, R. The next level, level 1, is the space of intervals. Level 2 is the space of fuzzy numbers (possibility distributions on R) and the space of random numbers (probability distributions on R). The top level, level 3, is the space of Z-numbers.

A Z-valuation is an ordered triple of the form (X,A,B) which is equivalent to the assignment of a Z-number (A,B) to X, written as X is (A,B) A collection of Z-valuations is referred to as Z-information. What is important to observe is that much of uncertain information in everyday experience is representable as Z-information. Example: Usually, Robert leaves office at about 5 pm. Usually, it takes Robert about an hour to get home from work. When does Robert get home? This information and the question may be represented as: (time of departure, about 5 pm, usually) and (time of travel, about l hour, usually); (time of arrival, ?A, ?B).

Computation with Z-numbers is complicated by the fact that what is known are not the underlying probability density functions but fuzzy restrictions on such functions. To deal with computation with fuzzy restrictions what is needed is the extension principle of fuzzy logic. Basically, the extension principle is a formalism for evaluating the value of a function when what are known are not the values of arguments but restrictions on the values of arguments.

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Computation with Z-numbers is an important generalization of computation with real numbers. In particular, the generality of Z-numbers opens the door to construction of better models of reality, especially in fields such as decision analysis, planning, risk assessment, economics and biomedicine.